MA244-12 Analysis III

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Jose Rodrigo
Credit value
12
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This covers three topics: (1) Riemann integration, (2) convergence of sequences and series of functions, (3) introduction to complex valued functions.
The idea behind integration is to compute the area under a curve. The fundamental theorem of calculus gives the precise relation between integration and differentiation. However, integration involves taking a limit, and the deeper properties of integration require a precise and careful analysis of this limiting process. This module proves that every continuous function can be integrated, and proves the fundamental theorem of calculus. It also discusses how integration can be applied to define some of the basic functions of analysis and to establish their fundamental properties.
Many functions can be written as limits of sequences of simpler functions (or as sums of series): thus a power series is a limit of polynomials, and a Fourier series is the sum of a trigonometric series with coefficients given by certain integrals. The second part of the module develops methods for deciding when a function defined as the limit of a sequence of other functions is continuous, differentiable, integrable, and for differentiating and integrating this limit. Norms are used at several stages and finally applied to show that a Differential Equation has a solution.
The final part of module focuses on complex valued functions, starting with the notion of complex differentiability. The module extends the results from Analysis II on power series to the complex case. The final section focuses on contour integrals, where a complex valued function is integrated along a curve. Cauchy's integral formula will be developped and a series of applications presented (to compute integrals of real valued functions, Liouville's Theorem and the Fundamental Theorem of Algebra).

[Module web page](#)

Module aims

To develop a good working knowledge of the construction of the Riemann integral in one variable;
To study the continuity, differentiability and integral of the limit of a uniformly convergent sequence of functions;
To extend the results from Analysis II on power series to the complex case.
To develop the notion of contour integration for complex valued functions, and explore several applications (Cauchy's integral formula, Liouville's Theorem, the fundamental theorem of algebra,...)

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Riemann Integration
    - Definition of the Riemann integral
      -Fundamental properties of the Riemann integral
    - The Fundamental Theorem of Calculus
    - Improper integrals
    - The Cantor Set and the devil’s staircase
- Sequences and Series of Functions
    - Pointwise and uniform convergence
    - Series of functions
    - A continuous, nowhere differentiable function
    - Space filling curves
    - Absolute Continuity
- Complex Analysis
    - Review of basic facts about C
    - Power Series
    - The exponential and the circular functions
    - Argument and Log
    - Complex integration, contour integrals
    - Links with MA259
    - Consequences of Cauchy’s Theorem
    - Applications of Cauchy’s formula to evaluate integrals in R

Learning outcomes

By the end of the module, students should be able to:

- To develop a good working knowledge of the construction of the Riemann integral;
- To understand the fundamental properties of the integral; main ones include: any continuous function can be integrated on a bounded interval and the Fundamental Theorem of Calculus (and its applications);
- To understand uniform and pointwise convergence of functions together with properties of

the limit function;
- To study the continuity, differentiability and integral of the limit of a uniformly convergent sequence of functions;
- To study complex differentiability (Cauchy-Riemann equations) and complex power series;
- To study contour integrals: Cauchy's integral formulas and applications.

Indicative reading list

- Lecture notes will be provided for the module.
- The module webpage contains additional references that students may consult.
  Students registered for this module may access the relevant chapters of books scanned under copyright.

Subject specific skills

- Working knowledge of the theory of Riemann integration.
- Theory for series and sequences, including the development of the notions of convergence and uniform converge for sequences and series of functions
- Working knowledge of complex analysis, to include power series, exponential and circular maps, contour integration.
- Applications of Cauchy's formula to compute integrals in R.

Transferable skills

- The students will be able to apply abstract notions in a variety of different contexts.
- Use a variety of techniques to compute complicated integrals or asymptotic expansions for functions/quantities arising from a wide range of applications in the physical sciences.
- Students will develop an ability to analyse and process complex information, triaging key concepts and effectively prepare plans for solving problems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (77%)
Tutorials	9 sessions of 1 hour (23%)
Total	39 hours

Private study description

81 hours private study, revision for exams, and assignments

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D2

	Weighting	Study time
Assignments	15%	
In-person Examination	85%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA244

Availability

Courses

This module is Core for:

- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe

This module is Core optional for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy

  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list B for:

- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)