MA222-12 Metric Spaces

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Richard Sharp
Credit value
12
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module builds on Analysis courses taken in Year 1 and Year 2. Here, the concepts of convergence and continuity are studied in the more general and more flexible setting of metric spaces and topological spaces. We also introduce and see the importance of the concepts of compactness, connectedness and completeness.

Module web page

Module aims

The module introduces the notions of normed space, metric space and topological space. In this setting we introduce open sets and closed sets, and discuss their relationship to convergence and continuity. We will introduce the concepts of compactness and connectedness, how they relate to continuity. We also consider completeness. This material provides the bridge from analysis on the real line, as considered in earlier analysis modules, to a much more flexible and general framework. This is essential for many later modules in Years 3 and 4, particular pure mathematics modules involving analysis, geometry or topology.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

To introduce the notions of Normed Space, Metric Space and Topological Space, and the fundamental properties of Compactness, Connectedness and Completeness that they may possess. Students will gain knowledge of definitions, theorems and calculations in

- Normed, Metric and Topological spaces
- Open and closed sets and their relation to continuity
- Notions of Compactness and relations to continuous maps
- Notions of Connectedness and relations to continuous maps
- Notions of Completeness and relations to previous topics in the module.
  The module comprises the following chapters:
- Normed Spaces
- Metric Spaces
- Open and closed sets
- Continuity
- Topological spaces
- Compactness
- Connectedness
- Completeness

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate understanding of the basic concepts, theorems and calculations of Normed, Metric and Topological Spaces.
- Demonstrate understanding of the open-set definition of continuity and its relation to previous notions of continuity, and applications to open or closed sets.
- Demonstrate understanding of the basic concepts, theorems and calculations of the concepts of Compactness, Connectedness and Completeness (CCC).
- Demonstrate understanding of the connections that arise between CCC, their relations under continuous maps, and simple applications.

Indicative reading list

1. W A Sutherland, Introduction to Metric and Topological Spaces, OUP.
2. E T Copson, Metric Spaces, CUP.
3. W Rudin, Principles of Mathematical Analysis, McGraw Hill.
4. G W Simmons, Introduction to Topology and Modern Analysis, McGraw Hill. (More advanced, although it starts at the beginning; helpful for several third year and MMath modules in analysis).
5. A M Gleason, Fundamentals of Abstract Analysis, Jones and Bartlett.

Subject specific skills

Students will develop understanding of metric and topological spaces, and of convergence and continuity in these settings. They will be able to characterise convergence and continuity in terms of open sets. They will have a good knowledge of a variety of examples of metric spaces and be able to determine whether or not they are topologically equivalent. They will be familiar with

topological spaces as a generalisation of metric spaces and understand the role of metrisability. They will understand the concept of compactness and how it is related to continuity. They will become familiar with the notion of connectedness and understand non-trivial examples. They will understand Cauchy sequences and completeness of metric spaces and understand how to construct the completion of a non-complete space.

Transferable skills

The module deals with abstract mathematical concepts, where examples may be removed from students' normal intuition. Therefore, students who have successfully completed the module will have developed their skills in reasoning in an abstract setting, for example from a set of axioms. They will have the background to carry of further study in the area, and to apply the techniques they have learned to various areas of analysis, geometry and topology. More generally, they will have had the opportunity to develop their analytic skills through the study of complex and abstract systems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (77%)
Tutorials	9 sessions of 1 hour (23%)
Total	39 hours

Private study description

81 hours to review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B

	Weighting	Study time
In-person Examination	100%	

	Weighting	Study time
~Platforms - AEP		

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA222

Availability

Courses

This module is Core optional for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Optional for:

- Year 2 of USTA-G305 Undergraduate Data Science (MSci) (with Intercalated Year)
- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 2 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)

This module is Option list B for:

- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list E for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)