MA209-6 Variational Principles

23/24

Department
Warwick Mathematics Institute
Level
Undergraduate Level 2
Module leader
Mario Micallef
Credit value
6
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

This module consists of a study of the mathematical techniques of variational methods, with applications to problems in physics and geometry. Critical point theory for functionals in finite dimensions is developed and extended to variational problems.

Module web page

Module aims

To introduce the calculus of variations and to see how central it is to the formulation and understanding of physical laws and to problems in geometry.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

This module is an introduction into mathematical techniques of variational methods, with applications to problems in Physics and Geometry. The basic problem in the calculus of variations is to minimise an integral which depends on a differentiable function and its derivatives. The

module covers the following topics: a brief revision of critical points in finite dimension, the mathematical set up of a variational problem, Euler-Lagrange equations for functionals of different types (including a derivation of these equations), a discussion of appropriate boundary conditions, first integrals of the Euler Lagrange equations, applications of variational principles to classical mechanics (including the least action principle) and optics (Fermat's principle). The theory is extended to constrained variational problems using Lagrange multipliers. The theory is illustrated by numerous examples.

Learning outcomes

By the end of the module, students should be able to:

- At the conclusion of the course you should be able to set up and solve minimisation problems with and without constraints, to derive Euler-Lagrange equations and appreciate how the laws of mechanics and geometrical problems involving least length and least area fit into this framework.

Indicative reading list

A useful and comprehensive introduction is:
R Weinstock, Calculus of Variations with Applications to Physics and Engineering, Dover, 1974.
Other useful texts are:
F Hildebrand, Methods of Applied Mathematics (2nd ed), Prentice Hall, 1965.
IM Gelfand & SV Fomin. Calculus of Variations, Prentice Hall, 1963.
The module will not, however, closely follow the syllabus of any book.

Subject specific skills

At the conclusion of the module the student should be able to set up and solve various minimisation problems with and without constraints, to derive Euler-Lagrange equations and appreciate how the laws of mechanics and geometrical optics, as well as some geometrical problems involving least length and least area, fit into this framework.

Transferable skills

The student will learn how the methods of mathematical analysis studied in the first and second year can be applied to model some simple real world phenomena such as the motion of a mechanical object or the shape of a ray of light. The student will see how mathematical methods can be used to restate some fundamental laws of Physics. The students will also learn some basics features of optimisation problems.

Study

Study time

Type	Required
Lectures	15 sessions of 1 hour (25%)
Private study	45 hours (75%)
Total	60 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B1

	Weighting	Study time	Eligible for self-certification
In-person Examination	100%		No

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Exam feedback.

Past exam papers for MA209

Availability

Courses

This module is Core for:

- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)

This module is Optional for:

- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Core option list B for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 3 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 2 of G106 Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe

This module is Option list B for:

- Year 2 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics