MA148-10 Vectors and Matrices

23/24

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 1

Module leader
: Oleg Kozlovski

Credit value
: 10

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

Many problems in maths and science are solved by reduction to a system of simultaneous linear equations in a number of variables. Even for problems which cannot be solved in this way, it is often possible to obtain an approximate solution by solving a system of simultaneous linear equations, giving the "best possible linear approximation".

The branch of maths treating simultaneous linear equations is called linear algebra. The module contains a theoretical algebraic core, whose main idea is that of a vector space and of a linear map from one vector space to another. It discusses the concepts of a basis in a vector space, the dimension of a vector space, the image and kernel of a linear map, the rank and nullity of a linear map, and the representation of a linear map by means of a matrix.

These theoretical ideas have many applications, which will be discussed in the module. These applications include:

Solutions of simultaneous linear equations. Properties of vectors. Properties of matrices, such as rank, row reduction, eigenvalues and eigenvectors. Properties of determinants and ways of calculating them.

Module aims

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Vector spaces: vector space over R, functions, polynomials, R^n, euclidean space, a subspace.
2. Bases: linear dependence and independence, spanning, existence of basis (sifting in a finitely spanned space), dimension, orthonormal basis, writing vectors in an orthonormal basis.
3. Linear maps: linear maps f:V-->W, examples, isomorphism of vector spaces, correspondence between matrices and linear maps, change of basis, row and column operations, solution of linear equations, kernel, image, rank, row rank and column rank, Smith normal form, rank-nullity theorem.
4. Linear transformations: linear maps f:V-->V, square matrices, determinants, Det(AB) = Det(A)Det(B), minors, cofactors, the adjoint matrix, the inverse of a matrix, determinant is a volume.
5. Diagonalisation: eigenvalues and eigenvectors, their geometric significance, 2x2 matrices (with diagonalisation over C), diagonalisation of matrices with distinct eigenvalues, diagonalisation of symmetric matrices (no proofs).
6. Linear maps on euclidean spaces.

Learning outcomes

By the end of the module, students should be able to:

- understand vector spaces, linear dependence and independence, bases and dimension
- master the concept of linear transformation
- be familiar with matrix manipulation, reduction of a matrix using row and column operations and be able to apply to finding solutions to linear equations
- be able to compute determinants for general n by n matrices
- master computation of eigenvalues and eigenvectors of matrices and their geometric significance
- get familiar with linear transformations between euclidean spaces

Indicative reading list

David Towers, Guide to Linear Algebra, Macmillan 1988.
Howard Anton, Elementary Linear Algebra, John Wiley and Sons, 1994.
Paul Halmos, Linear Algebra Problem Book, MAA, 1995.
G Strang, Linear Algebra and its Applications, 3rd ed, Harcourt Brace, 1988.

Subject specific skills

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (48%)
Online learning (independent)	9 sessions of 1 hour (21%)
Private study	13 hours (31%)
Total	42 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments	15%	20 hours
In-person Examination	85%	38 hours

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes. Exam feedback is given.

Past exam papers for MA148

Availability

Courses

This module is Core for:

- USTA-G302 Undergraduate Data Science
  - Year 1 of G302 Data Science
  - Year 1 of G302 Data Science
- Year 1 of USTA-G304 Undergraduate Data Science (MSci)
- Year 1 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 1 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 1 of Y602 Mathematics,Operational Research,Stats,Economics