PX276-7.5 Methods of Mathematical Physics

22/23

Department
: Physics

Level
: Undergraduate Level 2

Module leader
: Rudo Roemer

Credit value
: 7.5

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

The module starts with the theory of Fourier transforms and the Dirac delta function. Fourier transforms are used to represent functions on the whole real line using linear combinations of sines and cosines. Fourier transforms are a powerful tool in physics and applied mathematics. A Fourier transform will turn a linear differential equation with constant coefficients into a nice algebraic equation which is in general easier to solve.

The module explains why diffraction patterns in the far-field limit are the Fourier transforms of the "diffracting" object. It then looks at diffraction generally. The case of a repeated pattern of motifs illustrates beautifully one of the most important theorems in the business - the convolution theorem. The diffraction pattern is simply the product of the Fourier transform of repeated delta functions and the Fourier transform for a single copy of the motif. The module also introduces Lagrange multipliers, co-ordinate transformations and cartesian tensors illustrating them with examples of their use in physics.

[Module web page]

Module aims

To teach mathematical techniques needed by second, third and fourth year physics modules

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Fourier Series:
   Representation for function f(x) defined -L to L; mention of convergence issues; real and complex forms; differentiation, integration; periodic extensions
2. Fourier Transforms:
   Fourier series when L tends to infinity. Definition of Fourier transform and standard examples - Gaussian, exponential and Lorentzian.
   Domains of application: (Time t | frequency w), (Space x | wave vector k).
   Delta function and properties, Fourier's Theorem.
   Convolutions, example of instrument resolution, convolution theorem
3. Interference and diffraction phenomena:
   Interference and diffraction, the Huygens-Fresnel principle. Criteria for Fraunhofer and Fresnel
   diffraction. Fraunhofer diffraction for parallel light. Fourier relationship between an object and its diffraction pattern. Convolution theorem demonstrated by diffraction patterns. Fraunhofer diffraction for single, double and multiple slits. Fraunhofer diffraction at a circular aperture; the
   Airy disc. Image resolution, the Rayleigh criterion and other resolution limits. Fresnel diffraction,
   shadow edges and diffraction at a straight edge
4. Lagrange Multipliers:
   Variation of f(x,y) subject to g(x,y) = constant implies grad f parallel to grad g. Lagrange multipliers. Example of quadratic form
5. Vectors and Coordinate Transformations:
   Summation convention, Kronecker delta, permutation symbol and use for representing vector
   products. Revision of cartesian coordinate transformations. Diagonalizing quadratic forms
6. Tensors:
   Physical examples of tensors: mass, current, conductivity, electric field
7. Stokes’ Theorem (Worksheet):
   Line integrals, circulation; curl in Cartesians; statement and proof of Stokes’ theorem for triangulations; dependence on region of integration and vector field; gradient, irrotational, solenoidal and incompressible vector fields; applications drawn from electromagnetism, fluid dynamics, condensed matter physics, differential geometry

Learning outcomes

By the end of the module, students should be able to:

- Represent functions in terms of Fourier series and Fourier transforms
- Demonstrate understanding of diffraction and interference phenomena
- Minimise/maximise functions subject to constraints using Lagrange multipliers

- Express vectors in different coordinate systems, recognise some physical examples of tensors
- Derive, and apply in physical contexts, Stokes's theorem

Indicative reading list

KF Riley,MP Hobson and SJ Bence, Mathematical Methods for Physics and Engineering: a Comprehensive Guide, Wadsworth, H D Young and R A Freedman, University Physics 11th Edition, Pearson.

View reading list on Talis Aspire

Subject specific skills

Mathematical methods including: Fourier transforms and their application to describe diffraction, Lagrange multipliers.
Skills in modelling, reasoning, thinking

Transferable skills

Analytical, communication, problem-solving, self-study

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (27%)
Other activity	10 hours (13%)
Private study	45 hours (60%)
Total	75 hours

Private study description

Working through lecture notes, solving problems, wider reading, discussing with others taking the module, revising for exam, practising on past exam papers

Other activity description

10 example classes

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D3

	Weighting	Study time
Assessed Coursework	20%	
In-person Examination	80%	
Answer 2 questions		

- Answerbook Green (8 page)
- Students may use a calculator

Assessment group R1

	Weighting	Study time
In-person Examination - Resit	100%	
Answer 2 questions		

- Answerbook Pink (12 page)
- Students may use a calculator

Feedback on assessment

Personal tutors, group feedback

[Past exam papers for PX276]

Availability

Courses

This module is Core for:

- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)

    - Year 2 of GF13 Mathematics and Physics
    - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)
    - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list B for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 2 of G100 Mathematics
    - Year 2 of G100 Mathematics
    - Year 2 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 2 of G100 Mathematics
    - Year 2 of G103 Mathematics (MMath)
    - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year