MA4M3-15 Local Fields

22/23

Department
Warwick Mathematics Institute
Level
Undergraduate Level 4
Module leader
Credit value
15
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

The real numbers R are defined as the completion of the rational numbers Q in the usual metric. However, this metric is not that well-suited to arithmetic study; for example, the integers are discrete in R.

In number theory, one is often more interested in p-adic numbers Qp, the completion of Q in the p-adic metric. In the p-adic metric, a number is very close to zero if it is highly divisible by a prime p (for example, whilst 1,000,000,000 is 'large' in the usual metric, it is highly divisible by 2 and 5, so it is very small in the 2-adic and 5-adic metrics). The integers are not discrete in the p-adic metric (as e.g. one can arbitrarily approximate 0 by integers p-adically), so p-adic numbers are much better suited to arithmetic, and have accordingly become fundamental in number theory and arithmetic geometry.

The real and p-adic numbers are examples of local fields. This module will give an introduction to local fields, with an emphasis on the p-adic numbers/non-archimedean local fields, and describe some of their beautiful properties, including: the classification of local fields, Hensel's lemma and applications to solubility of polynomials, and extensions and Galois theory of local fields.

Primary resources will include the books 'Local Fields' by Serre and 'Local Fields' by Cassels.

The course will also treat some notable applications in number theory and arithmetic geometry, in particular the Kronecker—Weber theorem on abelian extensions of Q and the Hasse—Minkowski theorem on solubility of quadratic forms.

Module aims

To give students a grounding in the theory of local fields (e.g. the p-adic or real numbers) and their relationship with global fields (e.g. the rationals), and to gain insight into the use of local methods to solve global problems.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Valuations and absolute values, classification of the valuations on Q, discrete valuation rings
- The p-adic numbers Qp, the p-adic integers Zp, and p-adic expansions
- Hensel’s lemma and applications
- Extensions of local fields: inertia groups and Galois theory, the Kronecker—Weber theorem
- Inverse limits
- Local-Global principles, the Hasse—Minkowski theorem

Learning outcomes

By the end of the module, students should be able to:

- Explain the definition, basic properties and classification of valuations and local fields
- Understand inverse limits and the topology of the p-adic integers
- Use Hensel’s lemma to determine solubility of polynomial equations over local fields
- Use the Hasse—Minkowski theorem to determine solubility of rational quadratic forms
- Describe the Galois theory of local fields, including solubility of the Galois group and classification of abelian extensions

Interdisciplinary

There has been work on understanding physical phenomena when one replaces the real numbers with p-adic numbers, for example in the field of p-adic quantum mechanics. This module would provide a path into this area of modern physics.

Subject specific skills

Ability to apply local methods to derive global consequences in arithmetic
Ability to work with different number systems

Transferable skills

Ability to translate scientific ideas into mathematical language
Ability to communicate complex ideas and mathematical results clearly
Ability to analyse and solve abstract mathematical problems

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (100%)
Total	30 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time	Eligible for self-certification
Assignments	15%	20 hours	Yes (extension)
In-person Examination	85%	36 hours	No

Standard 3 hour written exam.

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

Standard 3 hour written exam.

- Answerbook Pink (12 page)

Feedback on assessment

Written feedback on the outcome of the exam.

Past exam papers for MA4M3

Availability

Courses

This module is Option list B for:

- Year 2 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)

This module is Option list C for:

- Year 3 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- Year 3 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe