MA4J8-15 Commutative Algebra II

22/23

Department
Warwick Mathematics Institute
Level
Undergraduate Level 4
Module leader
Miles Reid
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Commutative Algebra is the study of commutative rings, and their modules and ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory.

The resulting unification, where the same underlying algebraic structures arise both in geometry and in number theory, is one of the crowning glories of modern mathematics and plays a fundamental role in current work. A simple instance of this unification will already be familiar to anyone who has seen the strong parallel between the ring of integers Z and the polynomial ring k[X] over a field k: both are integral domains having division-with-remainder, and therefore a Euclidean algorithm. In either case, unique factorisation into primes and the treatement of ideals and modules in terms of localisation at primes follows by the same kind of arguments. The ring of integers of an algebraic number field used since the 19th century to solve problems in number theory runs in parallel with the geometry of algebraic curves and their function theory.

This module is a second course in the subject (following MA3G6), and will bring students to the frontiers of research in Algebra, Number Theory and Algebraic Geometry.

Module aims

The module is intended for Year 4 MMath students, and also MASt, MSc and beginning PhD students interested in Algebra, Number Theory, Algebraic Geometry and Complex Function theory. A solid background in the contents of MA3G6 Commutative Algebra is assumed. The material includes advanced topics in the algebra of commutative rings and their modules, together with areas of application.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Review of basics of commutative algebra.
Factorisation, discrete valuations rings and normal rings.
The Artin-Rees lemma for completions and power series methods.
Dimension theory, the Hilbert and Hilbert-Samuel function.
Systems of parameters and multiplicity.
Regular sequences, the Koszul complex, depth, Cohen-Macaulay and Gorenstein rings.
Characterisation of regular local rings.

Learning outcomes

By the end of the module, students should be able to:

- Develop a sophisticated command of the many facets of a major branch of algebra with important applications across the whole of mathematics.
- Apply advanced notions of commutative algebra such as localisation, completion, dimension, extension, regular sequences, Hilbert and Hilbert-Samuel series, to rings and their modules, both in theoretical arguments and in practical applications.

Indicative reading list

M.F. Atiyah, I.G. MacDonald, Introduction to Commutative Algebra. Addison-Wesley 1969; reprinted by Perseus 2000.
M. Reid, Undergraduate Commutative Algebra. CUP 1995.
D. Eisenbud, Commutative algebra with a view toward algebraic geometry. Springer 1995.
H. Matsumura, Commutative ring theory, CUP 1986.

Subject specific skills

Working knowledge of commutative algebra.
Ability to apply commutative algebra methods to a variety of problems. including those in Algebraic Geometry and Number Theory.

Transferable skills

Ability to translate scientific ideas into mathematical language.
Ability to communicate complex ideas and mathematical results clearly.

Ability to analyse and solve abstract mathematical problems.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (32%)
Private study	64 hours (68%)
Total	94 hours

Private study description

Working on assigned worksheets, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time	Eligible for self-certification
Assignments	15%	20 hours	No
In-person Examination	85%	36 hours	No

Standard 3 hour written exam.

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit	100%		No

Standard 3 hour written exam.

	Weighting	Study time	Eligible for self-certification

- Answerbook Gold (24 page)

Feedback on assessment

Written feedback on the outcome of the exam.

Past exam papers for MA4J8

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)

This module is Option list A for:

- Year 2 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 2 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list B for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 2 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year

    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe