# MA3H0-15 Numerical Analysis & PDE's

# 22/23

Department Warwick Mathematics Institute Level Undergraduate Level 3 Module leader Markus Kirkilionis Credit value 15 Module duration 10 weeks Assessment Multiple Study location University of Warwick main campus, Coventry

# Description

### Introductory description

This module addresses the mathematical theory of discretization of partial differential equations (PDEs) which is one of the most important aspects of modern applied mathematics. Because of the ubiquitous nature of PDE based mathematical models in biology, finance, physics, advanced materials and engineering much of mathematical analysis is devoted to their study. The complexity of the models means that finding formulae for solutions is impossible in most practical situations. This leads to the subject of computational PDEs. On the other hand, the understanding of numerical solution requires advanced mathematical analysis. A paradigm for modern applied mathematics is the synergy between analysis, modelling and computation. This course is an introduction to the numerical analysis of PDEs which is designed to emphasise the interaction between mathematical theory and numerical methods.

#### Module web page

#### Module aims

The aim of this module is to provide an introduction to the analysis and design of numerical methods for solving partial differential equations of elliptic, hyperbolic and parabolic type.

### **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Analysis and numerical analysis of two point boundary value problems.

Model finite difference methods and their analysis.

Variational formulation of elliptic PDEs; function spaces; Galerkin method; finite element method; examples of finite elements; error analysis.

### Learning outcomes

By the end of the module, students should be able to:

- be able to discretise an elliptic partial differential equation using finite element and finite difference methods,
- carry out stability and error analysis for the discrete approximation to elliptic, parabolic and hyperbolic equations in certain domains.
- Students who have successfully taken this module should be aware of the issues around the discretization of several different types of pdes,
- have a knowledge of the finite element and finite difference methods that are used for discretizing,

### Indicative reading list

Background reading:

Stig Larsson and Vidar Thomee, Partial differential equations with numerical methods, Springer Texts in Applied Mathematics Volume 45 (2005).

K W Morton and D F Mayers, Numerical solution of partial differential equations: an introduction Cambridge University Press Second edition (2005).

### Subject specific skills

- Discretisation of partial differential equations: the students will learn typical problems that can be encountered when discretising PDEs and the most common ways of fixing or avoiding them.
- In particular, they will study convergence and stability properties of various numerical schemes (these are concepts that appear in numerical analysis in general and should be useful in the future).
- The students will also learn different forms of discretising PDEs in space and time, as well as how to decide which discretisation to use depending on the PDE they consider.

### Transferable skills

- Problem solving and written communication
- Students will be provided Matlab and/or Python scripts to work on, which will give them the opportunity to improve their programming skills (Improving employability)
- Numerical solution of PDEs is a common required skill for applied mathematics researchers

# Study

# Study time

Туре	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

### Private study description

Review lectured material and work on set exercises.

### Costs

No further costs have been identified for this module.

# Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group D

	Weighting	Study time
Assignments	15%	
In-person Examination	85%	
3 hour examination - no books allowed		

• Answerbook Gold (24 page)

### Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
3 hour exam - no books allowed		

• Answerbook Gold (24 page)

#### Feedback on assessment

Exam feedback

Past exam papers for MA3H0

# Availability

### Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - $\circ~$  Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic

#### and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
  - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
  - Year 1 of G1PC Mathematics (Diploma plus MSc)
  - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - $_{\circ}~$  Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - $\circ~$  Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- Year 3 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)