MA3D1-15 Fluid Dynamics

22/23

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Shreyas Mandre
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

The lectures will provide a solid background in the mathematical description of fluid dynamics. They will cover the derivation of the conservation laws (mass, momentum, energy) that describe the dynamics of fluids and their application to a remarkable range of phenomena including water waves, sound propagation, atmospheric dynamics and aerodynamics. The focus will be on deriving approximate expressions using (usually) known mathematical techniques that yield analytic (as opposed to computational) solutions.

Module web page

Module aims

An important aim of the module is to provide an appreciation of the complexities and beauty of fluid motion. This will be highlighted in class using videos of the phenomena under consideration (usually available on YouTube).

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Mathematical modelling of fluid flow.
Specification of the flow by field variables; vorticity; stream function; strain tensor; stress tensor. Euler's equation. Navier-Stokes equation. Introduction of non-dimensional parameters.

Additional conservation laws.
Bernoulli's equations. Global conservation laws.

Vortex dynamics.
Kelvin's circulation theorem. Helmholtz theorems. Cauchy-Lagrange theorem. 3D vorticity equation, vortex lines, vortex tubes and vortex stretching.

2D flows. Flow in a pipe.
Shear flows. Jet flow by similarity solution. Round vortices.

Irrotational 2D flows & classical aerofoil theory.
Complex analysis methods in Fluid Dynamics. Blasius theorem. Zhukovskii lift theorem. Force on a cylinder and force on an aerofoil.

Rotating flows.
Navier-Stokes in a rotating frame. Rossby number. Taylor-Proudman theorem. Geophysical flow.

Boundary layers. Prandtl's boundary layer theory. Ekman boundary layer in rotating fluids.

Waves.
General theory of waves. Sound waves. Free-surface flows & surface tension. Gravity-capillary water waves.

Instabilities.
Rayleigh criterion. Orr-Sommerfeld equation. Kelvin-Helmholtz instability. Stability of parallel flows.

Learning outcomes

By the end of the module, students should be able to:

- - be able to understand the derivation of the equations of fluid dynamics
- - master a range of mathematical techniques that enable the approximate solution to the aforementioned equations
- - be able to interpret the meanings of these solutions in 'real life' problems

Indicative reading list

Strongly recommended texts:
D.J. Acheson, Elementary Fluid Dynamics, OUP. (Excellent text with derivations, examples and solutions)
S. Nazarenko, Fluid Dynamics via Examples and Solutions, Taylor and Francis. (Great source of questions and detailed solutions.)

Further Reading:
A.R. Paterson, A First Course in Fluid Dynamics, CUP. (Easier than Acheson.)
L.D. Landau and E.M. Livshitz, Fluid Mechanics, OUP. (A classic for those with a deep interest in fluid dynamics in modern physics.)

D.J. Tritton, Physical Fluid Dynamics , Oxford Science Publs. (The emphasis is on the physical phenomena and less on the mathematics.)

Subject specific skills

See learning outcomes

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group B1

	Weighting	Study time
In-person Examination	100%	

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Marked coursework and exam feedback.

Past exam papers for MA3D1

Availability

Courses

This module is Core for:

- Year 3 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)
  - Year 3 of FG31 Mathematics and Physics (MMathPhys)

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 4 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with

Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)
- Year 3 of UPXA-F303 Undergraduate Physics (MPhys)
- Year 3 of UPXA-F3FA Undergraduate Physics with Astrophysics (MPhys)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)
- Year 3 of UPXA-F303 Undergraduate Physics (MPhys)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)