MA3A6-15 Algebraic Number Theory

22/23

Department
Warwick Mathematics Institute
Level
Undergraduate Level 3
Module leader
Simon Rydin Myerson
Credit value
15
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

Algebraic number theory is the study of algebraic numbers and especially algebraic integers, which are the roots of monic polynomials with integer coefficients. We study ways in which properties of algebraic integers resemble or differ from the properties of ordinary integers, especially with regard to prime factorisation. As well as their intrinsic elegance, this theory of algebraic integers leads to methods for finding solutions of equations in ordinary integers, such as the Mordell equation and Pell equation. Historically, much of the theory was developed in attempts to solve Fermat's Last Theorem. The module combines substantial theoretical results, relying on algebraic tools such as rings, ideals and abelian groups, with calculations of specific examples.

Module web page

Module aims

To demonstrate that uniqueness of factorization into irreducibles can fail in rings of algebraic integers, but that it can be replaced by the uniqueness of factorization into prime ideals.
To introduce some geometric lattice-theoretic techniques and their applications to algebraic number theory.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The objects of study in MA3A6 are number fields (finite field extensions of the rational numbers) and rings of algebraic integers. After defining these objects and their basic properties, we focus on factorisation in rings of integers. Unique factorisation into irreducible elements often fails in these rings, but there is unique factorisation for ideals into prime ideals - this is one of the biggest theorems in the module. We use ideals to define the class group of a number field, a finite abelian group which measures how badly unique factorisation fails. We study methods for calculating the class group and the prime factorisation of ideals, and apply these tools to solve some Diophantine equations.

Learning outcomes

By the end of the module, students should be able to:

- be able to compute norms and discriminants and to use them to determine the integer rings in algabraic number fields;
- be able to factorize ideals into prime ideals in algebraic number fields in straightforward examples;
- understand the proof of Minkowski's Theorem on lattices, and be able to apply it, for example, to prove that all positive integers are the sum of four squares.

Indicative reading list

This module is based on the book Algebraic Number Theory and Fermat's Last Theorem, by I.N. Stewart and D.O. Tall, published by A.K. Peters (2001). The contents of the module forms a proper subset of the material in that book. (The earlier edition, published under the title Algebraic Number Theory, is also suitable.) Another recommended book is Algebraic Number Theory by F. Jarvis (Springer). More advanced books are A Brief Guide to Algebraic Number Theory, by H.P.F. Swinnerton-Dyer (LMS Student Texts # 50, CUP), or Algebraic Number Theory, by A. Fröhlich and M.J. Taylor (CUP).

Subject specific skills

In this module, students will acquire a foundation for further study in number theory and for understanding its applications in cryptography. They will also develop skills to link abstract mathematical theory with computation in special cases.

Transferable skills

In this module, students improve their problem solving and analytical skills, with an emphasis on applying theoretical knowledge in computational settings. Students also develop their skills in clearly communicating logical arguments.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D1

	Weighting	Study time
Assignments	15%	
In-person Examination	85%	

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA3A6

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
    - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G4 Undergraduate Discrete Mathematics (with Intercalated Year)
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
    - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
    - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
    - Year 4 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 1 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
    - Year 2 of G1PD Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
    - Year 1 of G1PC Mathematics (Diploma plus MSc)
    - Year 2 of G1PC Mathematics (Diploma plus MSc)

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 4 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
  - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 5 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
  - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
  - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)

- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)