

# MA251-12 Algebra I: Advanced Linear Algebra

**22/23**

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 2

**Module leader**

Adam Thomas

**Credit value**

12

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

This module is a continuation of First Year Linear Algebra.

[Module web page](#)

### Module aims

To develop further and to continue the study of linear algebra, which was begun in Year 1 and to point out and briefly discuss applications of the techniques developed to other branches of mathematics, physics, etc.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

This module is a continuation of First Year Linear Algebra. In that course we studied conditions under which a matrix is similar to a diagonal matrix, but we did not develop methods for testing whether two general matrices are similar. Our first aim is to fill this gap for matrices over  $\mathbb{C}$ . Not all

matrices are similar to a diagonal matrix, but they are all similar to one in Jordan canonical form; that is, to a matrix which is almost diagonal, but may have some entries equal to 1 on the superdiagonal.

We next study quadratic forms. A quadratic form is a homogeneous quadratic expression in several variables. Quadratic forms occur in geometry as the equation of a quadratic cone, or as the leading term of the equation of a plane conic or a quadric hypersurface. By a change of coordinates, we can always write in the diagonal form  $q(x)$ . For a quadratic form over  $R$ , the number of positive or negative diagonal coefficients is an invariant of the quadratic form which is very important in applications.

Finally, we study matrices over the integers, and investigate what happens when we restrict methods of linear algebra, such as elementary row and column operations, to operations over  $Z$ . This leads, perhaps unexpectedly, to a complete classification of finitely generated abelian groups.

## **Learning outcomes**

By the end of the module, students should be able to:

- By the end of the module students should be familiar with: the theory and computation of the the Jordan canonical form of matrices and linear maps; bilinear forms, quadratic forms, and choosing canonical bases for these; the theory and computation of the Smith normal form for matrices over the integers, and its application to finitely generated abelian groups.

## **Indicative reading list**

P M Cohn, Algebra, Vol. 1, Wiley

I N Herstein, Topics in Algebra, Wiley.

## **Subject specific skills**

This module teaches students how to carry out a number of fundamental calculations with matrices, including the theory and computation of the Jordan canonical form of matrices and linear maps; bilinear forms, diagonalizing quadratic forms, and choosing canonical bases for these; the theory and computation of the Smith normal form for matrices over the integers, and its application to finitely generated abelian groups.

## **Transferable skills**

The algorithmic techniques taught have widespread "real world" applications. Examples include ranking in search engines, linear programming and optimisation, signal analysis, and graphics. To also include: clear and precise thinking; the ability to follow complex reasoning; constructing logical arguments, and exposing illogical ones; and formulating problems as algorithms, thereby enhancing understanding of details and rendering them suitable for computer implementation.

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## **Study**

## Study time

Type	Required
Lectures	30 sessions of 1 hour (25%)
Tutorials	9 sessions of 1 hour (8%)
Other activity	10 hours (8%)
Private study	71 hours (59%)
Total	120 hours

## Private study description

Review lectured material and work on set exercises.

## Other activity description

Collaborative project

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

## Assessment group D2

	Weighting	Study time
Assignment	15%	
Five assignments given		
In-person Examination	85%	
2 hour exam, no books allowed		

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- Answerbook Pink (12 page)

## Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
<ul style="list-style-type: none"> <li>• Answerbook Pink (12 page)</li> </ul>		

## Assessment group S

	Weighting	Study time
Assignment	15%	
In-person Examination	85%	
<ul style="list-style-type: none"> <li>• Answerbook Pink (12 page)</li> </ul>		

## Feedback on assessment

Marked assignments and exam feedback

[Past exam papers for MA251](#)

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## Availability

## Courses

This module is Core for:

- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy

- Year 2 of GV17 Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations
- Year 2 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Core optional for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year

This module is Optional for:

- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 2 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 2 of GG14 Mathematics and Statistics
  - Year 2 of GG14 Mathematics and Statistics
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Option list B for:

- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 2 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 3 of GG14 Mathematics and Statistics
  - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 3 of Y602 Mathematics, Operational Research, Stats, Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics, Operational Research, Statistics, Economics (with Intercalated Year)

This module is Option list E for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated
  - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
  - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)