MA151-10 Algebra 1

22/23

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 1

Module leader
: Dmitriy Rumynin

Credit value
: 10

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

This module introduces important algebraic structures including groups, rings and fields. Students will learn how to verify that a set is a group, ring or field, and how to carry out elementary operations in these structures. They will also assimilate permutations, symmetric groups, and alternating groups, and know how to determine the unit group of a ring.

Module aims

The main idea is to teach Algebra “naturally”, similarly to the way children learn school-level Algebra. According to this, abstractions should come after calculations. Hence, as far as abstractions are concerned, we are going back to High School. For instance, Isomorphism Theorems, including Orbit-Stabiliser Theorem, should wait until Algebra-3. So is Lagrange’s Theorem (except the abelian groups version that has a quick proof).

Our approach is not to be confused with “Harvard Calculus” from 1990-s: we give complete definitions and state theorems precisely in this module.

It is important to get interaction with Foundations. In week 7 the following material is required for Foundations: fields, groups, the multiplicative group, little Fermat theorem, Euler’s totient function. On the other hand, Chinese Remainder Theorem and RSA are to be covered in Foundations.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Group Theory: motivating examples (numbers, cyclic group, dihedral group, symmetric group, transformations of the plane), elementary properties, subgroups, Lagrange's Theorem for abelian groups, odd and even permutations.
- Ring Theory: commutative and non-commutative rings, fields, examples (Z[x], Z/nZ, F[x], F[x]/(f)), unit groups, factorisations in Z and polynomials.
- List of covered algebraic definitions: group, subgroup, group homomorphism (including kernel, image, isomorphism), order, sign of permutation, ring, field, subring, ideal, ring homomorphism (including kernel, image, isomorphism), quotient ring

Learning outcomes

By the end of the module, students should be able to:

- understand the abstract definition of a group and a group homomorphism
- be familiar with the dihedral and cyclic groups as well as the group of euclidean transformations of the plane
- perform manipulations with the elements of the symmetric group, representing them as a product of compositions
- understand the orders of elements as well as the proof of Lagrange’s Theorem for abelian groups
- get the working knowledge of the understand the definition of various types of ring, and be familiar with a number of examples, including numbers, polynomials and Z/nZ
- perform manipulations with polynomials over Z, R and C
- learn the unit groups of rings, in particular, of Z/nZ

Indicative reading list

Samir Siksek, Introduction to Abstract Algebra lecture notes,
Ronald Solomon, Abstract Algebra, Brooks/Cole, 2003.
Niels Lauritzen, Concrete Abstract Algebra, Cambridge University Press, 2003

Subject specific skills

This module introduces important algebraic structures including groups, rings and fields. Students will learn how to verify that a set is a group, ring or field, and how to carry out elementary operations in these structures. They will understand the relation between a group, a subgroup and the cosets of a subgroup which leads to Lagrange's theorem. They will also assimilate permutations, symmetric groups, and alternating groups, and know how to determine the unit group of a ring.

Transferable skills

The module reinforces logical thinking and deductive reasoning which are valuable transferable skills. The algebraic structures introduced are the heart of modern cryptography and information security.

Study

Study time

Type	Required
Lectures	20 sessions of 1 hour (20%)
Online learning (independent)	9 sessions of 1 hour (9%)
Private study	13 hours (13%)
Assessment	58 hours (58%)
Total	100 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Assignments homeworks	15%	20 hours
In-person Examination final exam	85%	38 hours

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
final resit exam		

- Answerbook Pink (12 page)

Feedback on assessment

Marked homework (both assessed and formative) is returned and discussed in smaller classes. Exam feedback is given.

[Past exam papers for MA151]

Availability

Courses

This module is Core for:

- Year 1 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 1 of G100 Mathematics
  - Year 1 of G103 Mathematics (MMath)
  - Year 1 of G103 Mathematics (MMath)
- Year 1 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 1 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 1 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 1 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 1 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 1 of UMAA-G101 Undergraduate Mathematics with Intercalated Year