MA138-12 Sets and Numbers

22/23

Department
> Warwick Mathematics Institute

Level
> Undergraduate Level 1

Module leader
> Saul Schleimer

Credit value
> 12

Assessment
> Multiple

Study location
> University of Warwick main campus, Coventry

Description

Introductory description

Mathematics can be described as the science of logical deduction - if we assume such and such as given, what can we deduce with absolute certainty? Consequently mathematics has a very high standard of truth - the only way to establish a mathematical claim is to give a complete, rigorous proof. Sets and Numbers aims to show students what can be achieved through abstract mathematical reasoning.

[Module web page](#)

Module aims

University mathematics introduces progressively more and more abstract ideas and structures, and demands more and more in the way of proof, until by the end of a mathematics degree most of the student's time is occupied with understanding proofs and creating his or her own. This is not because university mathematicians are more pedantic than schoolteachers, but because proof is how one knows things in mathematics, and it is in its proofs that the strength and richness of mathematics is to be found. But learning to deal with abstraction and with proofs takes time. This module aims to bridge the gap between school and university mathematics, by beginning with some rather concrete techniques where the emphasis is on calculation, and gradually moving towards abstraction and proof.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Numbers:
   Number systems: Natural numbers, integers, rationals and real numbers. Existence of irrational numbers.
   Euclidean algorithm; greatest common divisor and least common multiple.
   Prime numbers, existence and uniqueness of prime factorisation (and non-uniqueness in other "number systems", e.g. even integers, Gaussian integers).
   Properties of commutativity, associativity and distributivity.
   Infinity of the primes.
   Summing series of integers; proofs by induction.

2. Language:
   Basic set theory: Intersection, Union, Venn diagrams and de Morgan's Laws.
   Logical connectives and, or, implies and their relation with intersection and union

3. Polynomials:
   Multiplication and long division of polynomials.
   Binomial theorem
   Euclidean algorithm for polynomials.
   Remainder theorem; a degree n polynomial has at most n roots.
   Rational functions and partial fractions.
   Incompleteness of the real numbers, completeness of the complex numbers (sketch).

4. Counting:
   Elementary combinatorics as practice in bijections, injections and surjections.
   Cardinality of the set of subsets of a set X is greater than cardinality of X.
   Russell's paradox.
   Definition of Cartesian product.
   Countability of the rational numbers, uncountability of the reals.
   Transcendental numbers exist!

The second (and smaller) part of the module explores the elementary properties of a fundamental algebraic structure called a group. Groups arise in an extraordinary range of contexts in mathematics and beyond (for example, in elementary particle physics and in card tricks), and can be used to analyse the symmetry of geometric objects or physical systems.

1. Modular arithmetic: 3 hours:
   Addition, multiplication and division in the integers modulo n.
   Some theorems of modular arithmetic.
   Equivalence relations.

2. Permutations and the symmetric group:
   Multiplying (composing) permutations.
   Cycles and disjoint cycle representation.
   The sign of a permutation.
   Basic Group Theory

Learning outcomes

By the end of the module, students should be able to:

- Work with number systems and develop fluency with their properties
- Learn the language of sets and quantifiers, of functions and relations
- Become familiar with various methods and styles of proof

Indicative reading list

None of these is the course text, but each would be useful, especially the first:
A.F.Beardon, Algebra and Geometry, CUP, 2005.
I.N. Stewart and D.O. Tall, Foundations of Mathematics, OUP, 1977.
J. A. Green, Sets and Groups; First Course in Algebra, Chapman and Hall, 1995.

Subject specific skills

Sets and Numbers will provide students with an introduction both to the language and to the methodology of university level mathematics. By studying familiar objects, but from a deeper and more rigorous perspective, they will become accustomed to constructing and evaluating logical arguments, as well as learn how to communicate these arguments precisely. These skills will be used throughout the rest of their degree.

Transferable skills

Critical thinking, problem solving and analytical skills, group work.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (88%)
Tutorials	8 sessions of 30 minutes (12%)
Total	34 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D2

	Weighting	Study time
Tests multiple choice tests	15%	
Online Examination exam	85%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit Exam	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Verbal feedback in supervisions, fortnightly multiple choice tests.

[Past exam papers for MA138](#)

Availability

Courses

This module is Core for:

- Year 1 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)