MA132-10 Foundations

22/23

Department
Warwick Mathematics Institute
Level
Undergraduate Level 1
Module leader
David Wood
Credit value
10
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

University mathematics introduces progressively more and more abstract ideas and structures, and demands more and more in the way of proof, until by the end of a mathematics degree most of the student's time is occupied with understanding proofs and creating his or her own. This is not because university mathematicians are more pedantic than schoolteachers, but because proof is how one knows things in mathematics, and it is in its proofs that the strength and richness of mathematics is to be found.

Module web page

Module aims

But learning to deal with abstraction and with proofs takes time. This module aims to bridge the gap between school and university mathematics, by beginning with some rather concrete techniques where the emphasis is on calculation, and gradually moving towards abstraction and proof.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Number systems
Number systems: Natural numbers, integers. Rationals and real numbers. Existence of irrational numbers. Complex Numbers.
Polar and exponential form of complex numbers. De Moivre's Theorem, n'th roots and roots of unity.
Euclidean algorithm; greatest common divisor and least common multiple.
Prime numbers, existence and uniqueness of prime factorisation. Infiniteness of the set of primes.
Modular arithmetic. Congruence, addition and multiplication modulo
n.
Language and Proof
Proof by induction.
Well-ordering Principle.
Proof by contradiction.
Basic set theory: $\cap$,$\cup$, Venn diagrams and de Morgan's Laws. Cartesian product of sets, power set.
Logical connectives$\wedge$, $\vee$, $\Rightarrow$ and their relation with $\cap$, $\cup$and $\subseteq$. Quantifiers $\forall$ and $\exists$.
Sets, functions and relations
Injective, surjective and bijective functions.
Inverse functions.
Relations: equivalence relations, order relations.
Polynomials
Multiplication and long division of polynomials.
Euclidean algorithm for polynomials.
Remainder theorem; a degree n polynomial has at most n roots.
Algebraic and transcendental numbers. Fundamental theorem of
Algebra (statement only).
Counting
Cardinalities, including infinite cardinalities.
Cardinality of the power set of X is greater than cardinality of X.
Russell's paradox.
Countability of the rational numbers, uncountability of the reals.
Transcendental numbers exist!

Learning outcomes

By the end of the module, students should be able to:

- Students will work with number systems and develop fluency with their properties;
- they will learn the language of sets and quantifiers, of functions and relations, and will become familiar with various methods and styles of proof.

Indicative reading list

None of these is the course text, but each would be useful, especially the first:
A.F.Beardon, Algebra and Geometry, CUP, 2005.
I.N. Stewart and D.O. Tall, Foundations of Mathematics, OUP, 1977.
J. A. Green, Sets and Groups; First Course in Algebra, Chapman and Hall, 1995.

Subject specific skills

After taking this module, students will be familiar with the concept of a rigorous mathematical proof and most of commonly used notations.

Transferable skills

Students will work with number systems and develop fluency with their properties; they will learn the language of sets and quantifiers, of functions and relations, and will become familiar with various methods and styles of proof.

Study

Study time

Type	Required
Lectures	28 sessions of 1 hour (28%)
Tutorials	8 sessions of 30 minutes (4%)
Private study	68 hours (68%)
Total	100 hours

Private study description

Reviewing lectured material and revising for 10 weekly assignments with 5 fortnightly tests based on them.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
4 Tests Multiple choice tests	15%	
In-person Examination	85%	

	Weighting	Study time
exam		

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit exam	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Tests will be marked and feedback given after exam.

Past exam papers for MA132

Availability

Courses

This module is Core for:

- Year 1 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 1 of G100 Mathematics
  - Year 1 of G103 Mathematics (MMath)
  - Year 1 of G103 Mathematics (MMath)
- Year 1 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 1 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 1 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 1 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 1 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)

- Year 1 of UMAA-G101 Undergraduate Mathematics with Intercalated Year