# MA106-12 Linear Algebra

## 22/23

**Department**
    Warwick Mathematics Institute
**Level**
    Undergraduate Level 1
**Module leader**
    Diane Maclagan
**Credit value**
    12
**Assessment**
    Multiple
**Study location**
    University of Warwick main campus, Coventry

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## Description

### Introductory description

Many problems in maths and science are solved by reduction to a system of simultaneous linear equations in a number of variables. Even for problems which cannot be solved in this way, it is often possible to obtain an approximate solution by solving a system of simultaneous linear equations, giving the "best possible linear approximation".

The branch of maths treating simultaneous linear equations is called linear algebra. The module contains a theoretical algebraic core, whose main idea is that of a vector space and of a linear map from one vector space to another. It discusses the concepts of a basis in a vector space, the dimension of a vector space, the image and kernel of a linear map, the rank and nullity of a linear map, and the representation of a linear map by means of a matrix.

These theoretical ideas have many applications, which will be discussed in the module. These applications include:

Solutions of simultaneous linear equations.
Properties of vectors.
Properties of matrices, such as rank, row reduction, eigenvalues and eigenvectors.
Properties of determinants and ways of calculating them.

Module web page

### Module aims

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

## Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- The vector space R^n, including a geometric description of vector addition in R^2.
- Fields. Definition of a vector space V over a field. The space spanned by a subset of V. Linear dependence and independence. Bases. Dimension. Subspaces. Dual spaces and dual bases.
- Linear maps f:V->W. Isomorphism of vector spaces. Any n-dimensional vector space over F is isomorphic to R^n. Examples of linear maps, including differentiation and integration as linear maps on spaces of functions or polynomials.
- Matrices. Algebraic operations on matrices. Reduction of a matrix using row and column operations. Application to the solution of linear equations. Rank. Row rank = Column rank.
- The relation between linear maps and matrices. the matrix of a linear map with respect to a given basis. Change of basis changes A to PAQ^{-1}. The kernal and image of f:V->W. The rank and nullity of f.
- Determinants, defined by ∑σϵSn sign σ(∏ai,σ(i)). Det(AB) = Det(A)Det(B) (proof either in general or in the cases n=1,2,3). Submatrices, minors, cofactors, the adjoint matrix. Rules for calculating determinants. The inverse of a matrix. Ax=0 has non-zero solution if and only if det(A)=0. Determinantal rank.
- Eigenvalues and eigenvectors. Definition and examples. Their geometric significance. Diagonalisation of matrices with distinct eigenvalues.
- Inner product spaces and isometries. Euclidean spaces. Orthogonal transformations and matrices.

## Learning outcomes

By the end of the module, students should be able to:

- Understand and demonstrate knowledge of vector spaces, fields, linear dependence and independence, bases and dimension.
- Understand linear transformations and be able to show examples of linear maps such as differentiation and integration as linear maps on spaces of functions of polynomials.
- Be proficient at matrix manipulation, reduction of a matrix using row and column operations and be able to apply to finding solutions to linear equations.
- Be able to compute determinants for general n by n matrices, compute cofactors and adjoint matrices and understand the implications of doing this to solving sets of linear equations.
- Be able to compute eigenvalues and eigenvectors of matrices and understand their geometric significance. Be able to diagonalize matrices with distinct eigenvalues.

## Indicative reading list

David Towers, Guide to Linear Algebra, Macmillan 1988.

Howard Anton, Elementary Linear Algebra, John Wiley and Sons, 1994.
Paul Halmos, Linear Algebra Problem Book, MAA, 1995.
G Strang, Linear Algebra and its Applications, 3rd ed, Harcourt Brace, 1988.

### Subject specific skills

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

### Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

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## Study

## Study time

| Type | Required |
| --- | --- |
| Lectures | 30 sessions of 1 hour (91%) |
| Tutorials | 6 sessions of 30 minutes (9%) |
| Total | 33 hours |

### Private study description

Working on assignments, going over lecture notes, text books, exam revision.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group D2

| | Weighting | Study time |
| --- | --- | --- |
| Assessment (non-Maths students) | 15% | |

| | Weighting | Study time |
|---|---|---|
| weekly, summative, assignments | | |
| In-person Examination<br>Exam | 85% | |

- Answerbook Pink (12 page)

**Assessment group R**

| | Weighting | Study time |
|---|---|---|
| In-person Examination - Resit<br>exam | 100% | |

- Answerbook Pink (12 page)

**Feedback on assessment**

Marked assignments, face to face supervisions.

[Past exam papers for MA106](#)

## Availability

## Courses

This module is Core for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 1 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)