MA106-12 Linear Algebra

22/23

Department
    Warwick Mathematics Institute
Level
    Undergraduate Level 1
Module leader
    Diane Maclagan
Credit value
    12
Assessment
    Multiple
Study location
    University of Warwick main campus, Coventry

Description

Introductory description

Many problems in maths and science are solved by reduction to a system of simultaneous linear equations in a number of variables. Even for problems which cannot be solved in this way, it is often possible to obtain an approximate solution by solving a system of simultaneous linear equations, giving the "best possible linear approximation".

The branch of maths treating simultaneous linear equations is called linear algebra. The module contains a theoretical algebraic core, whose main idea is that of a vector space and of a linear map from one vector space to another. It discusses the concepts of a basis in a vector space, the dimension of a vector space, the image and kernel of a linear map, the rank and nullity of a linear map, and the representation of a linear map by means of a matrix.

These theoretical ideas have many applications, which will be discussed in the module. These applications include:

Solutions of simultaneous linear equations.
Properties of vectors.
Properties of matrices, such as rank, row reduction, eigenvalues and eigenvectors.
Properties of determinants and ways of calculating them.

Module web page

Module aims

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- The vector space R^n, including a geometric description of vector addition in R^2.
- Fields. Definition of a vector space V over a field. The space spanned by a subset of V. Linear dependence and independence. Bases. Dimension. Subspaces. Dual spaces and dual bases.
- Linear maps f:V->W. Isomorphism of vector spaces. Any n-dimensional vector space over F is isomorphic to R^n. Examples of linear maps, including differentiation and integration as linear maps on spaces of functions or polynomials.
- Matrices. Algebraic operations on matrices. Reduction of a matrix using row and column operations. Application to the solution of linear equations. Rank. Row rank = Column rank.
- The relation between linear maps and matrices. the matrix of a linear map with respect to a given basis. Change of basis changes A to PAQ^{-1}. The kernal and image of f:V->W. The rank and nullity of f.
- Determinants, defined by ∑σϵSn sign σ(∏ai,σ(i)). Det(AB) = Det(A)Det(B) (proof either in general or in the cases n=1,2,3). Submatrices, minors, cofactors, the adjoint matrix. Rules for calculating determinants. The inverse of a matrix. Ax=0 has non-zero solution if and only if det(A)=0. Determinantal rank.
- Eigenvalues and eigenvectors. Definition and examples. Their geometric significance. Diagonalisation of matrices with distinct eigenvalues.
- Inner product spaces and isometries. Euclidean spaces. Orthogonal transformations and matrices.

Learning outcomes

By the end of the module, students should be able to:

- Understand and demonstrate knowledge of vector spaces, fields, linear dependence and independence, bases and dimension.
- Understand linear transformations and be able to show examples of linear maps such as differentiation and integration as linear maps on spaces of functions of polynomials.
- Be proficient at matrix manipulation, reduction of a matrix using row and column operations and be able to apply to finding solutions to linear equations.
- Be able to compute determinants for general n by n matrices, compute cofactors and adjoint matrices and understand the implications of doing this to solving sets of linear equations.
- Be able to compute eigenvalues and eigenvectors of matrices and understand their geometric significance. Be able to diagonalize matrices with distinct eigenvalues.

Indicative reading list

David Towers, Guide to Linear Algebra, Macmillan 1988.

Howard Anton, Elementary Linear Algebra, John Wiley and Sons, 1994.
Paul Halmos, Linear Algebra Problem Book, MAA, 1995.
G Strang, Linear Algebra and its Applications, 3rd ed, Harcourt Brace, 1988.

Subject specific skills

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (91%)
Tutorials	6 sessions of 30 minutes (9%)
Total	33 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D2

	Weighting	Study time
Assessment (non-Maths students)	15%	

	Weighting	Study time
weekly, summative, assignments		
In-person Examination Exam	85%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit exam	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments, face to face supervisions.

[Past exam papers for MA106]

Availability

Courses

This module is Core for:

- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 1 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)