PX275-15 Mathematical Methods for Physicists

21/22

Department
: Physics

Level
: Undergraduate Level 2

Module leader
: Gary Barker

Credit value
: 15

Module duration
: 20 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

The module reviews the techniques of ordinary and partial differentiation and ordinary and multiple integration. It develops vector calculus and discusses the partial differential equations of physics (Term 1). The theory of Fourier transforms and the Dirac delta function are also covered. Fourier transforms are used to represent functions on the whole real line using linear combinations of sines and cosines. Fourier transforms are a powerful tool in physics and applied mathematics. The examples used to illustrate the module are drawn mainly from interference and diffraction phenomena in optics (Term 2).

Module web page

Module aims

To teach mathematical techniques needed by second, third and fourth year physics modules.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Revision: functions of more than one variable, partial differentiation, chain rule, Taylor series. Change of coordinates for functions of more than one variable. Revision of vectors, the cross product and relationship of its modulus to parallelogram area. Vector areas for the context of surface integrals.
2. Multiple Integrals: Line, surface and volume integrals. Length of curves, surfaces of revolution. Change of variable and change of order.
3. Vector differentiation: Scalar and vector fields. Mathematical definition of grad and its physical meaning - examples. The divergence defined mathematically and a physical interpretation. Relationship with flux. The Laplacian, Solenoidal fields. Physics examples.
4. Gauss's Divergence Theorem: Demonstration of its validity for rectangular and cylindrical volumes. Examples.
5. Stokes’s Theorem: The curl and its interpretation. Conservative fields, irrotational fields. Stokes’s theorem and its derivation. Examples.
6. PDEs: The wave equation, Poisson's equation, Schrödinger's equation. The diffusion equation and Fick's law. The role of boundary conditions. Separation of variables. Examples.
7. Fourier Series (revision): Representation for function f(x) defined -L to L; brief mention of convergence issues; real and complex forms; differentiation, integration; periodic extension
8. Fourier Transforms:
   Fourier series when L tends to infinity. Definition of Fourier transform and standard examples: Gaussian, exponential and Lorentzian.
   Domains of application: (Time t | frequency w), (Space x | wave vector k).
   Delta function and properties, Fourier's Theorem.
   Convolutions, example of instrument resolution, convolution theorem.
9. Interference and diffraction phenomena:
   Interference and diffraction: the Huygens-Fresnel principle. Criteria for Fraunhofer and Fresnel diffraction. Fraunhofer diffraction for parallel light. Fourier relationship between an object and its diffraction pattern. Convolution theorem demonstrated by diffraction patterns. Fraunhofer diffraction for single, double and multiple slits. Fraunhofer diffraction at a circular aperture; the Airy disc. Image resolution, the Rayleigh criterion and other resolution limits. Fresnel diffraction, shadow edges and diffraction at a straight edge.

Learning outcomes

By the end of the module, students should be able to:

- Deal with multiple integrals and know how to evaluate the length of a curve and the volume of a three dimensional object
- Define and calculate the gradient, divergence and curl of a vector field and understand Gauss’s and Stokes’ theorems
- Define a partial differential equation and solve the wave and diffusion equations using the method of separation of variables
- Represent simple functions using Fourier transforms
- Demonstrate a good understanding of diffraction and interference phenomena and solve problems involving Fraunhofer diffraction

Indicative reading list

KF Riley,MP Hobson and SJ Bence, Mathematical Methods for Physics and Engineering: a Comprehensive Guide, Wadsworth, H D Young and R A Freedman, University Physics 11th Edition, Pearson.

[View reading list on Talis Aspire]

Subject specific skills

Mathematical methods including: Vector calculus, separation of variables, Fourier transforms and their application to describe diffraction

Transferable skills

Analytical, communication, problem-solving, self-study

Study

Study time

Type	Required
Lectures	40 sessions of 1 hour (27%)
Other activity	20 hours (13%)
Private study	90 hours (60%)
Total	150 hours

Private study description

Working though lecture notes, solving problems, wider reading, discussing with others taking the module, revising for exam, practising on past exam papers

Other activity description

20 Example classes

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time
Class Tests and Assessed Coursework Class Tests/Assessed Coursework	20%	
On-campus Examination Answer 4 questions	80%	

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Pink (12 page)

Feedback on assessment

Personal tutors, group feedback

Past exam papers for PX275

Availability

Courses

This module is Core for:

- Year 2 of UPXA-F304 Undergraduate Physics (BSc MPhys)
- Year 2 of UPXA-F300 Undergraduate Physics (BSc)
- Year 2 of UPXA-F303 Undergraduate Physics (MPhys)
- Year 2 of UPXA-F3N1 Undergraduate Physics and Business Studies
- Year 2 of UPXA-F3F5 Undergraduate Physics with Astrophysics (BSc)
- Year 2 of UPXA-F3FA Undergraduate Physics with Astrophysics (MPhys)
- Year 2 of UPXA-F3N2 Undergraduate Physics with Business Studies