

# PH210-15 Logic II : Metatheory

**21/22**

**Department**

Philosophy

**Level**

Undergraduate Level 2

**Module leader**

Benedict Eastaugh

**Credit value**

15

**Module duration**

10 weeks

**Assessment**

100% exam

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

This module will develop the metatheory of propositional and first-order logic. Our primary goal will be to show that a proof system similar to that presented in Logic I is sound (i.e. proves only logically true sentences) and complete (proves all logically true sentences). In order to better understand how we prove things about (as opposed to within) a proof system, we will first study the syntax, semantics, and proof theory of propositional logic. We will then consider Tarski's definitions of satisfaction and truth in a model and proceed to develop the Henkin completeness proof for first-order logic. Other topics covered along the way will include countable versus uncountable sets, the compactness theorem, and the expressive limitations of first-order logic. PH210 is recommended as a prerequisite for PH340 (Logic III: Incompleteness and Undecidability), PH341 (Modal Logic), and PH345 (Computability Theory).

### Module aims

To expose students to the basic metalogical notions of soundness and completeness. A natural deduction system for propositional and first order logic will be introduced and proven to be sound and complete. Along the way, basic mathematical tools needed for proving these results will be developed. These include elementary set theory and inductive proofs and definitions.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- Week 1: introduction, set theory, inductive definitions
- Week 2: syntax and semantics of propositional logic
- Week 3: natural deduction
- Week 4: soundness of propositional logic
- Week 5: completeness of propositional logic
- Week 7: syntax and semantics of first-order logic
- Week 8: natural deduction and soundness for first-order logic
- Week 9: completeness for first-order logic
- Week 10: the compactness theorem and applications

## **Learning outcomes**

By the end of the module, students should be able to:

- demonstrate knowledge of the Soundness and Completeness Theorems for propositional and first-order logic and related technical results and definitions (subject knowledge and understanding)
- understand the significance these concepts and results have for logic and mathematics (cognitive skills)
- use and define concepts with precision, both within formal and discursive contexts (key skills)
- write precise mathematical proofs (subject specific skills)

## **Indicative reading list**

Our primary text will be a version of the Open Logic text customised for PH210.

The same material is also covered in a number of other sources including:

Logic and Structure, 5th edition by Dirk van Dalen, Springer Verlag, 2008.

Much of the material we will be covering is also presented at a more elementary level in chapters 15–19 of the textbook for PH136 (Logic I):

Language, Proof, and Logic by Jon Barwise and John Etchemendy, CSLI Publications, 2002.

Students desiring additional background on problem solving techniques are also encouraged to obtain:

How to Prove It by Daniel J. Velleman, Cambridge University Press, 2006.

[View reading list on Talis Aspire](#)

## **Subject specific skills**

Demonstrate knowledge of the Soundness and Completeness Theorems for propositional and first-order logic and related technical results and definitions

Understand the significance these concepts and results have for logic and mathematics  
Use and define concepts with precision, both within formal and discursive contexts

## Transferable skills

Understand definitions and proof techniques from metalogic applicable in other domains -- e.g. such as mathematical and structural induction, basic set theoretic operations and constructions  
The ability to comprehend and construct precise mathematical proofs"

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## Study

### Study time

Type	Required
Lectures	9 sessions of 3 hours (18%)
Seminars	9 sessions of 1 hour (6%)
Private study	114 hours (76%)
Total	150 hours

### Private study description

No private study requirements defined for this module.

## Costs

No further costs have been identified for this module.

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## Assessment

You must pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group B6

	Weighting	Study time
On-campus Examination	100%	

### Feedback on assessment

Discussion and feedback on exercises during seminar.

[Past exam papers for PH210](#)

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## Availability

### Courses

This module is Core for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
  - Year 2 of GV17 Mathematics and Philosophy
- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 2 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Optional for:

- UECA-LM1D Undergraduate Economics, Politics and International Studies
  - Year 2 of LM1D Economics, Politics and International Studies
  - Year 2 of LM1D Economics, Politics and International Studies
- UHIA-V1V8 Undergraduate History and Philosophy (with Year Abroad and a term in Venice)
  - Year 3 of V1V8 History and Philosophy (with Year Abroad and a term in Venice)
  - Year 4 of V1V8 History and Philosophy (with Year Abroad and a term in Venice)
- Year 3 of UHIA-V1V7 Undergraduate History and Philosophy (with a term in Venice)
- UPHA-V700 Undergraduate Philosophy
  - Year 2 of V700 Philosophy
  - Year 2 of V700 Philosophy
  - Year 3 of V700 Philosophy
  - Year 3 of V700 Philosophy
- Year 4 of UPHA-V701 Undergraduate Philosophy (with Intercalated year)
- Year 4 of UPHA-V702 Undergraduate Philosophy (with Work Placement)
- UPHA-VQ72 Undergraduate Philosophy and Literature
  - Year 2 of VQ72 Philosophy and Literature
  - Year 3 of VQ72 Philosophy and Literature
- UPHA-V7ML Undergraduate Philosophy, Politics and Economics
  - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
  - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
  - Year 2 of V7ML Philosophy, Politics and Economics (Tripartite)
- UPHA-V7MM Undergraduate Philosophy, Politics and Economics (with Intercalated year)
  - Year 4 of V7MQ Philosophy, Politics and Economics (Bipartite) with Intercalated Year
  - Year 4 of V7MH Philosophy, Politics and Economics - Economics/Philosophy Bipartite (Economics Major) (with Intercalated year)
  - Year 4 of V7MF Philosophy, Politics and Economics - Economics/Politics Bipartite (Economics Major) (with Intercalated year)

- Year 4 of V7MI Philosophy, Politics and Economics - Philosophy/Economics Bipartite (Philosophy Major) (with Intercalated year)
- Year 4 of V7MJ Philosophy, Politics and Economics - Philosophy/Politics Bipartite (with Intercalated year)
- Year 4 of V7MG Philosophy, Politics and Economics - Politics/Economics Bipartite (Politics Major) (with Intercalated year)

This module is Option list A for:

- UPHA-VL78 BA in Philosophy with Psychology
  - Year 2 of VL78 Philosophy with Psychology
  - Year 3 of VL78 Philosophy with Psychology
- Year 4 of UPHA-VL79 BA in Philosophy with Psychology (with Intercalated year)

This module is Option list B for:

- Year 2 of UHIA-V1V5 Undergraduate History and Philosophy
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 2 of G105 Mathematics (MMath) with Intercalated Year
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 2 of G106 Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UMAA-G101 Undergraduate Mathematics with Intercalated Year
  - Year 2 of G101 Mathematics with Intercalated Year
  - Year 4 of G101 Mathematics with Intercalated Year
- UPHA-VQ72 Undergraduate Philosophy and Literature
  - Year 2 of VQ72 Philosophy and Literature

- Year 3 of VQ72 Philosophy and Literature
- Year 4 of UPHA-VQ73 Undergraduate Philosophy and Literature with Intercalated Year

This module is Option list C for:

- Year 3 of UHIA-V1V5 Undergraduate History and Philosophy
- Year 4 of UHIA-V1V6 Undergraduate History and Philosophy (with Year Abroad)

This module is Option list E for:

- UPHA-V7MW Undergraduate Politics, Philosophy and Law
  - Year 2 of V7MW Politics, Philosophy and Law
  - Year 2 of V7MW Politics, Philosophy and Law