# MA3J4-15 Mathematical Modelling with PDE

## 21/22

**Department**
    Warwick Mathematics Institute
**Level**
    Undergraduate Level 3
**Module leader**
    Marie-Therese Wolfram
**Credit value**
    15
**Assessment**
    Multiple
**Study location**
    University of Warwick main campus, Coventry

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## Description

### Introductory description

The students will be given a general overview on the derivation and use of partial differential equations modelling real world applications. By the end of the course they should have acquired knowledge about the physical interpretation of PDE models and how the learned techniques can be applied to similar problems.

[Module web page]

### Module aims

The module focuses on mathematical modelling with the help of PDEs and the general concepts and techniques behind it. It gives an introduction to PDE modelling in general and provides the necessary basics.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Mathematical modelling
   Math. modelling in physics, chemistry, biology, medicine, economy, finance, art, transport,

architecture, sports
Qualitative/quantitative models, discrete/continuum models
Scaling, dimensionless variables, sensitivity analysis
Examples: projectile motion, chemical reactions

2. Diffusion and drift
   Microscopic derivation
   Continuity equation and Fick's law
   Heat equation: scaling, properties of solutions
   Reaction diffusion systems: Turing instabilities
   Fokker-Planck equation

3. Transport and flows
   Conservation of mass, momentum and energy
   Euler and Navier-Stokes equations

4.From Newton to Boltzmann
Newton's laws of motion
Vlasov and Boltzmann equation
Traffic flow models

## Learning outcomes

By the end of the module, students should be able to:

- Understand the nature of micro- and macroscopic models.
- Formulate models in dimensionless quantities
- Have an overview of well known PDE models in physics and continuum mechanics
- Calculate solutions for simple PDE models
- Use and adapt Matlab programs provided during the module

## Indicative reading list

J. David Logan, Applied Mathematics: A Contemporary Approach
C.C.Lin, A. Segel, Mathematics Applied to Deterministic Problems in the Natural Sciences, 1988
A. Aw, A. Klar, Rascle and T Materne, Derivation of continuum traffic flow models from microscopic follow the leader models, SIAM Appl Math., 2002
B. Perthame, Transport equations in biology, Birkhäuser, 2007
R. Illner, Mathematical Modelling: A Case Study Approach, SIAM, 2005
C. Eck, H. Garcke, P. Knaber, Mathematische Modellierung, Springer, 2008
L. Pareschi and G. Toscani, Interacting Multiagent Systems, Oxford University Press, 2013

## Subject specific skills

- Mathematical modelling in the life and social sciences: the module introduces different modelling concepts and discusses the relation of these models across different scales.

- Understand and apply fundamental physical concepts, such as conservation of mass, ... to develop mathematical models.

- PDE specific techniques: the students will acquire different analytic techniques, such as characteristic calculus, entropy and energy methods, linear stability analysis, scaling techniques, ... These methods allow them to analyse the behaviour of solutions to the discussed PDE models in different situations (long time behaviour, scaling limits, ....)
- Construct solutions using the method of characteristics, fundamental solutions, separation of variables or scaling invariances.
- Analyse the behaviour of solutions in the presence of boundary conditions and obtain a better understanding of the qualitative behaviour of solutions.
- Familiarize themselves with Python and Matlab programs

### Transferable skills

- Model development: the module will introduce the students to the underlying concepts of PDE modelling. These techniques can be used and further developed in a more general context.
- PDE techniques: apply and develop well-known concepts in PDE analysis to study the solutions of more general PDEs.
- The Python and Matlab scripts provided give students the opportunity to improve the programming skills.
- The scripts can be improved and generalised for more complex or different PDE models
- Analytical thinking and problem solving: The course trains students to identify the main driving forces in applications and develop suitable mathematical models. Mathematical models play an important role in engineering, economics and finance. Being able to develop models and analyse them using analytical and computational techniques increases student possibilities on the job market. Mathematical modelling always comes with a certain degree of freedom - the course provides the students with the necessary tools to explore these freedom and gives them the freedom to develop them further.

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## Study

## Study time

| Type | Required |
|---|---|
| Lectures | 30 sessions of 1 hour (77%) |
| Tutorials | 9 sessions of 1 hour (23%) |
| Total | 39 hours |

### Private study description

Review lectured material and work on set exercises.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

### Assessment group B

| | Weighting | Study time |
|---|---|---|
| In-person Examination | 100% | |

- Answerbook Gold (24 page)

### Assessment group R

| | Weighting | Study time |
|---|---|---|
| In-person Examination - Resit | 100% | |

- Answerbook Gold (24 page)

### Feedback on assessment

Marked coursework and exam feedback.

Past exam papers for MA3J4

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## Availability

## Courses

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy

    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 3 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- Year 3 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year