# MA106-12 Linear Algebra

## 21/22

Department Warwick Mathematics Institute Level Undergraduate Level 1 Module leader Diane Maclagan Credit value 12 Module duration 10 weeks Assessment Multiple Study location University of Warwick main campus, Coventry

# Description

#### Introductory description

Many problems in maths and science are solved by reduction to a system of simultaneous linear equations in a number of variables. Even for problems which cannot be solved in this way, it is often possible to obtain an approximate solution by solving a system of simultaneous linear equations, giving the "best possible linear approximation".

The branch of maths treating simultaneous linear equations is called linear algebra. The module contains a theoretical algebraic core, whose main idea is that of a vector space and of a linear map from one vector space to another. It discusses the concepts of a basis in a vector space, the dimension of a vector space, the image and kernel of a linear map, the rank and nullity of a linear map, and the representation of a linear map by means of a matrix.

These theoretical ideas have many applications, which will be discussed in the module. These applications include:

Solutions of simultaneous linear equations.

Properties of vectors.

Properties of matrices, such as rank, row reduction, eigenvalues and eigenvectors. Properties of determinants and ways of calculating them.

Module web page

## Module aims

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

## **Outline syllabus**

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- The vector space R^n, including a geometric description of vector addition in R^2.
- Fields. Definition of a vector space V over a field. The space spanned by a subset of V. Linear dependence and independence. Bases. Dimension. Subspaces. Dual spaces and dual bases.
- Linear maps f:V->W. Isomorphism of vector spaces. Any n-dimensional vector space over F is isomorphic to R^n. Examples of linear maps, including differentiation and integration as linear maps on spaces of functions or polynomials.
- Matrices. Algebraic operations on matrices. Reduction of a matrix using row and column operations. Application to the solution of linear equations. Rank. Row rank = Column rank.
- The relation between linear maps and matrices. the matrix of a linear map with respect to a given basis. Change of basis changes A to PAQ^{-1}. The kernal and image of f:V->W. The rank and nullity of f.
- Determinants, defined by ∑σ∈Sn sign σ(∏ai,σ(i)). Det(AB) = Det(A)Det(B) (proof either in general or in the cases n=1,2,3). Submatrices, minors, cofactors, the adjoint matrix. Rules for calculating determinants. The inverse of a matrix. Ax=0 has non-zero solution if and only if det(A)=0. Determinantal rank.
- Eigenvalues and eigenvectors. Definition and examples. Their geometric significance. Diagonalisation of matrices with distinct eigenvalues.
- Inner product spaces and isometries. Euclidean spaces. Orthogonal transformations and matrices.

## Learning outcomes

By the end of the module, students should be able to:

- Understand and demonstrate knowledge of vector spaces, fields, linear dependence and independence, bases and dimension.
- Understand linear transformations and be able to show examples of linear maps such as differentiation and integration as linear maps on spaces of functions of polynomials.
- Be proficient at matrix manipulation, reduction of a matrix using row and column operations and be able to apply to finding solutions to linear equations.
- Be able to compute determinants for general n by n matrices, compute cofactors and adjoint matrices and understand the implications of doing this to solving sets of linear equations.
- Be able to compute eigenvalues and eigenvectors of matrices and understand their geometric significance. Be able to diagonalize matrices with distinct eigenvalues.

#### Indicative reading list

David Towers, Guide to Linear Algebra, Macmillan 1988. Howard Anton, Elementary Linear Algebra, John Wiley and Sons, 1994. Paul Halmos, Linear Algebra Problem Book, MAA, 1995. G Strang, Linear Algebra and its Applications, 3rd ed, Harcourt Brace, 1988.

#### Subject specific skills

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

## Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

## Study

## Study time

Туре	Required
Lectures	30 sessions of 1 hour (25%)
Tutorials	6 sessions of 30 minutes (2%)
Private study	87 hours (72%)
Total	120 hours

#### Private study description

Working on assignments, going over lecture notes, text books, exam revision.

## Costs

No further costs have been identified for this module.

## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

## Assessment group D2

	Weighting	Study time
Assessment (non-Maths students)	15%	
weekly, summative, assignments		
In-person Examination	85%	
Exam		

• Answerbook Pink (12 page)

#### Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
exam		

Answerbook Pink (12 page)

#### Feedback on assessment

Marked assignments, face to face supervisions.

Past exam papers for MA106

# Availability

#### **Post-requisite modules**

If you pass this module, you can take:

• MA398-15 Matrix Analysis & Algorithms

## Courses

This module is Core for:

- USTA-G302 Undergraduate Data Science
  - Year 1 of G302 Data Science

- Year 1 of G302 Data Science
- Year 1 of USTA-G304 Undergraduate Data Science (MSci)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 1 of G4G1 Discrete Mathematics
  - Year 1 of G4G1 Discrete Mathematics
- Year 1 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 1 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
  - Year 1 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 1 of G100 Mathematics
  - Year 1 of G103 Mathematics (MMath)
  - Year 1 of G103 Mathematics (MMath)
- Year 1 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 1 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 1 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 1 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 1 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 1 of GV17 Mathematics and Philosophy
  - Year 1 of GV17 Mathematics and Philosophy
  - Year 1 of GV17 Mathematics and Philosophy
- UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
  - Year 1 of GV18 Mathematics and Philosophy with Intercalated Year
- Year 1 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 1 of GF13 Mathematics and Physics
  - Year 1 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 1 of FG31 Mathematics and Physics (MMathPhys)
  - Year 1 of FG31 Mathematics and Physics (MMathPhys)
- Year 1 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 1 of GG14 Mathematics and Statistics
  - Year 1 of GG14 Mathematics and Statistics
- Year 1 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 1 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 1 of Y602 Mathematics, Operational Research, Stats, Economics