PX922-15 Approximation Theory for PDEs and Machine Learning

20/21

Department
Physics
Level
Taught Postgraduate Level
Module leader
Nicholas Hine
Credit value
15
Module duration
10 weeks
Assessment
60% coursework, 40% exam
Study location
University of Warwick main campus, Coventry

Description

Introductory description

N/A.

Module web page

Module aims

The Module will provide students with a foundation in approximation theory, driven by its applications in scientific computing and data science. In approximation theory a function that is difficult or impossible to evaluate directly, e.g., an unknown constitutive law or the solution of a PDE, is to be approximated as efficiently as possible from a more elementary class of functions, the approximation space, such as global polynomials, trigonometric polynomials (plane-waves), splines, radial basis functions, ridge functions. The module will provide students with the theoretical understanding and the tools to choose appropriate approximation tools in different applications chosen from typical scientific computing and data science as well as methods to construct the approximations, e.g., interpolation, least-squares, Gaussian process.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Part 1: univariate approximation

- spline approximation of smooth functions in 1D
- polynomial and trigonometric approximation of analytic functions in 1D
- linear best approximation
- best n-term approximation
- multi-variate approximation by tensor products in Rd, curse of dimensionality
  Part 2: Multi-variate approximation: details will depend on the progress through Part 1 and available time, but the idea of Part 2 is to cover a few selected examples of high-dimensional approximation theory, for example a sub-set of the following:
- mixed regularity, splines and sparse grids, Smolyak algorithm
- radial basis functions and Gaussian processes
- ridge functions and neural networks
- compressed sensing and best n-term approximation
  Throughout the lecture each topic will cover (1) approximation rates, (2) algorithms, and (3) examples, typically implemented in Julia or Python.

Learning outcomes

By the end of the module, students should be able to:

- Understand the key concepts, theorems and calculations of univariate and low-dimensional multi-variate approximation theory.
- Understanding a selection of the basic concepts, theorems and calculations of high-dimensional approximation theory.
- Understand the basic algorithms and examples used in approximation theory.
- Appreciate how the choice of approximation space affects the numerical error in the numerical solution of PDEs, and the connections between (high-dimensional) approximation theory and machine learning.

Subject specific skills

Understand the key concepts, theorems and calculations of univariate and low-dimensional multi-variate approximation theory
Understand a selection of the basic concepts, theorems and calculations of high-dimensional approximation theory
Understand the basic algorithms and examples used in approximation theory
Appreciate how the choice of approximation space affects the numerical error in the numerical solution of PDEs, and the connections between (high-dimensional) approximation theory and machine learning.

Transferable skills

Mathematics, Programming, Oral presentation

Study

Study time

Type	Required
Lectures	15 sessions of 2 hours (20%)
Practical classes	3 sessions of 1 hour (2%)
Private study	82 hours (55%)
Assessment	35 hours (23%)
Total	150 hours

Private study description

Reading etc

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D

	Weighting	Study time
Machine learning workshop exercises Based on the machine learning workshop exercises	20%	10 hours
Uncertainty propagation exercise Based on the uncertainty propagation workshop.	20%	10 hours
Predictive multiscale modelling exercise Based on predictive multiscale modelling.	20%	10 hours
Viva Voce examination 30 Minutes.	40%	5 hours

Feedback on assessment

Written annotations to submitted computational notebooks \r\nVerbal discussion during viva voce exam \r\nWritten summary of viva performance

Past exam papers for PX922

Availability

Courses

This module is Core optional for:

- Year 1 of TPXA-F344 Postgraduate Taught Modelling of Heterogeneous Systems