PH342-15 Philosophy of Mathematics

20/21

Department
> Philosophy

Level
> Undergraduate Level 3

Credit value
> 15

Module duration
> 10 weeks

Assessment
> Multiple

Study location
> University of Warwick main campus, Coventry

Description

Introductory description

Do mathematical objects such as numbers and sets exist or are they merely useful fictions? What is the nature of mathematical knowledge and how is it distinct from our knowledge of the physical world? What, if any, is the connection between the two? What role does mathematics play in the empirical sciences? What is the correct logic for reasoning about mathematics? Are formally undecidable statements (e.g. the Parallel Postulate, the Gödel sentence, the Continuum Hypothesis) objectively true or false? This module will explore different ways in which philosophy might be of help in answering these questions, both from the contemporary perspective and that of the major foundational schools of the late nineteen and early twentieth centuries -- i.e. logicism, intuitionism, and formalism.

Module aims

This module has two goals:

1. to familiarise students with major developments in the foundations of mathematics from the late 19th century onward;
2. to illustrate how these developments inform contemporary debates in philosophy of mathematics.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Indicative schedule and summary of readings
Lecture 1: Introduction, philosophical terminology

- Shapiro chapters 1, 2
- Benacerraf "Mathematical truth"

Lecture 2 : Classical perspectives: Plato & Aristotle

- Shapiro chapter 3
- Plato selections from the Republic and the Meno
- Aristotle selections from Metaphysics M

Lecture 3: Rationalism and empiricism: Kant & Mill

- Shapiro chapter 4
- Kant selections from Prolegomena
- Mill selections from A system of logic

Lecture 4: Logic and set theory interlude/review

- George and Velleman chapter 3
- Set Theory (SEP)
- Second-order and Higher-order Logic (SEP)
- Some formal theories of arithmetic (SEP)

Lecture 5: Logicism: Frege & Russell

- Shapiro chapter 5
- Frege selections from The foundations of arithmetic
- Heck "Frege's Theorem: An Introduction"
- "Frege's logic, theorem, and foundations for arithmetic" [SEP]

Lecture 6: Intuitionism: Brouwer & Heyting

- Shapiro chapter 6
- Heyting "Disputation" [BP]
- George and Velleman chapters 4 and 5
- Optional: Dummett "The philosophical basis of intuitionistic logic"

Lecture 7: Formalism and the Hilbert Programme

- Shapiro chapter 7
- George and Velleman chapter 6
- Hilbert "On the infinite" [BP]
- Zach "Hilbert's program" [SEP]

Lecture 8: Paradoxes, G¨odel's Theorems, and set theoretic independence

- Shapiro chapter 8
- Russell's Paradox [SEP]
- G¨odel's Incompleteness Theorems [SEP]
- The Continuum Hypothesis [SEP]
- Optional: G¨odel "What is Cantor's Continuum Hypothesis?", Dummett "The philosophical significance of G¨odel's Theorem"

Lecture 9: Structuralism

- Shapiro chapters 9,10
- Structuralism and Nominalism [SEP]
- Benacerraf "What numbers could not be" [BP]

Learning outcomes

By the end of the module, students should be able to:

- Demonstrate knowledge of some of the central topics in the philosophy of mathematics, and of the historical development of key approaches to the philosophy of mathematics (Subject knowledge and understanding)
- Understand the significance questions in the philosophy of mathematics have to wider issues in philosophy and the foundations of mathematics (cognitive skills)
- Articulate their own view of the relative merits of different theories and engage critically with the arguments put forward in support of them (key skills)
- Show an understanding of methodological issues in the philosophy of mathematics, and of questions of demarcation between philosophy and mathematics (subject-specific skills)

Indicative reading list

Our primary texts will be

- Thinking about mathematics, Stewart Shapiro, Oxford University Press, 2000.
- Philosophies of mathematics, Alexander George and David Velleman, Wiley-Blackwell, 2001.
  Many of the other reading are available in
- Philosophy of Mathematics: Selected Readings edited by Paul Benacerraf and Hilary Putnam,
  Cambridge University Press, 1983. [BP]
- The Stanford Encyclopedia of Philosophy [SEP].
  Other sources are available through the Moodle page.

Subject specific skills

Show an understanding of methodological issues in the philosophy of mathematics, and of questions of demarcation between philosophy and mathematics

Transferable skills

Understand how major debates in the philosophy of mathematics -- e.g. between logicism, formalism, and intuitionism -- are related to topics in the history of philosophy, metaphysics, and epistemology. Appreciate how developments in mathematical logic -- e.g. axiomatic set theory, proof theory -- grew out of concern for foundational issues in the 19th and early 20th century.

Study

Study time

Type	Required
Lectures	9 sessions of 2 hours (12%)
Seminars	8 sessions of 1 hour (5%)
Private study	124 hours (83%)
Total	150 hours

Private study description

No private study requirements defined for this module.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D5

Assessment component	Weighting	Study time	Eligible for self-certification
1000 word essay	20%		Yes (extension)

	Weighting	**Study time**	**Eligible for self-certification**

Reassessment component is the same

Assessment component

Online Examination 2 hour exam	80%		No

- Online examination: No Answerbook required

Reassessment component is the same

Assessment group D6

	Weighting	**Study time**	**Eligible for self-certification**

Assessment component

1000 word essay	20%		Yes (extension)

Reassessment component is the same

Assessment component

Online Examination	80%		No

- Online examination: No Answerbook required

Reassessment component is the same

Assessment group D7

	Weighting	**Study time**	**Eligible for self-certification**

Assessment component

	Weighting	Study time	Eligible for self-certification
1000 word essay	20%		Yes (extension)

Reassessment component is the same

Assessment component

Online Examination	80%		No

- Online examination: No Answerbook required

Reassessment component is the same

Feedback on assessment

Written feedback on essays and exams.

[Past exam papers for PH342](#)

Availability

Pre-requisites

PH136 (Logic 1) is recommended as a prerequisite. Otherwise, the module is designed to be as self-contained as possible. But you should be aware that several of the topics we will discuss are related to developments in mathematical logic (as treated in modules like Logic II/III and Set Theory) and also build on philosophical themes which are covered in modules like Metaphysics and Epistemology. So background in these subjects will be helpful for fully engaging with the module content.

To take this module, you must have passed:

- All of

  - [PH136-15 Logic 1: Introduction to Symbolic Logic](#)

Courses

This module is Core for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Optional for:

- UPHA-V700 Undergraduate Philosophy
  - Year 2 of V700 Philosophy
  - Year 3 of V700 Philosophy
- Year 4 of UPHA-V701 Undergraduate Philosophy (wiith Intercalated year)
- Year 4 of UPHA-V7MM Undergraduate Philosophy, Politics and Economics (with Intercalated year)

This module is Core option list A for:

- Year 3 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list B for:

- Year 2 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UPHA-VL78 BA in Philosophy with Psychology
  - Year 2 of VL78 Philosophy with Psychology
  - Year 3 of VL78 Philosophy with Psychology
- Year 4 of UPHA-VL79 BA in Philosophy with Psychology (with Intercalated year)

This module is Option list B for:

- Year 2 of UHIA-V1V5 Undergraduate History and Philosophy
- Year 2 of UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
- Year 2 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- UMAA-G101 Undergraduate Mathematics with Intercalated Year
  - Year 2 of G101 Mathematics with Intercalated Year
  - Year 4 of G101 Mathematics with Intercalated Year
- UPHA-VQ72 Undergraduate Philosophy and Literature

- Year 2 of VQ72 Philosophy and Literature
- Year 3 of VQ72 Philosophy and Literature

This module is Option list C for:

- Year 3 of UHIA-V1V5 Undergraduate History and Philosophy
- Year 4 of UHIA-V1V6 Undergraduate History and Philosophy (with Year Abroad)