

# MA4L2-15 Statistical Mechanics

20/21

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 4

**Module leader**

Daniel Ueltschi

**Credit value**

15

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

This will be an introduction to statistical mechanics, that deals with large systems of interacting particles. There will be a thorough review of the Ising model and its properties, including a mathematical description of its magnetic phase transition. The second part of the module will be spent on other models and other questions, such as systems with continuous symmetry; gaussian model; infinite-volumes Gibbs states; quantum spin systems.

[Module web page](#)

### Module aims

To familiarise students with statistical mechanics models, phase transitions, and critical behaviour.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Statistical mechanics describes physical systems with a huge number of particles. In physics, the goal is to describe macroscopic phenomena in terms of microscopic models and to give a

meaning to notions such as temperature or entropy. Mathematically, it can be viewed as the study of random variables with spatial dependence. Models of statistical mechanics form the background for recent advances in probability theory and stochastic analysis, such as SLE and the theory of regularity structures. So, they form an important background for understanding these topics of modern mathematics.

The module will give a thorough mathematical introduction to the Ising model and to the gaussian free field on regular graphs, and to the theory of infinite volume Gibbs measures.

## Learning outcomes

By the end of the module, students should be able to:

- Apply basic ideas of phase transitions and critical behaviour to lattice systems of statistical mechanics
- Understand the theory of infinite volume Gibbs measures
- Understand how large complex systems at equilibrium can be described from microscopic rules
- Have understood basic ideas of phase transitions and critical behaviour in the case of the Ising model and the gaussian free field; they will have mastered the theory of infinite volume Gibbs measures.

## Indicative reading list

We will mainly follow Chapter 3 of the new introductory textbook:

Sacha Friedli and Yvan Velenik, *Equilibrium Statistical Mechanics of Classical Lattice Systems: a Concrete Introduction*. Available at: <http://www.unige.ch/mth/folks/velenik/smbook/index.html>

Interested students can also look in

David Ruelle, *Statistical Mechanics: Rigorous Results*, World Scientific, 1999.

James Sethna: *Statistical Mechanics: Entropy, Order Parameters and Complexity* Oxford Master Series in Physics, 2006.

## Subject specific skills

We will review methods that allow to derive the macroscopic properties of large systems of interacting particles. We will do this in the specific case of models of statistical mechanics at equilibrium, but this is conceptually more general.

## Transferable skills

Understand and deal with large systems of interacting agents.

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## Study

## Study time

| Type          | Required                    |
|---------------|-----------------------------|
| Lectures      | 30 sessions of 1 hour (20%) |
| Tutorials     | 9 sessions of 1 hour (6%)   |
| Private study | 111 hours (74%)             |
| Total         | 150 hours                   |

## Private study description

Private study, assessed work, revision for exam.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

### Assessment group B1

|                       | Weighting | Study time | Eligible for self-certification |
|-----------------------|-----------|------------|---------------------------------|
| In-person Examination | 100%      |            | No                              |

- Answerbook Gold (24 page)

### Assessment group R

|                               | Weighting | Study time | Eligible for self-certification |
|-------------------------------|-----------|------------|---------------------------------|
| In-person Examination - Resit | 100%      |            | No                              |

- Answerbook Gold (24 page)

## Feedback on assessment

Marked coursework and exam feedback.

[Past exam papers for MA4L2](#)

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# Availability

## Courses

This module is Optional for:

- TMAA-G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
  - Year 1 of G1PE Master of Advanced Study in Mathematical Sciences
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
  - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
  - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- Year 4 of UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)

This module is Option list B for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- Year 4 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Option list C for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
  - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G103 Undergraduate Mathematics (MMath)

- Year 3 of G103 Mathematics (MMath)
- Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
  - Year 4 of G106 Mathematics (MMath) with Study in Europe

This module is Option list E for:

- Year 4 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- Year 5 of USTA-G301 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics (with Intercalated