MA3J3-15 Bifurcations, Catastrophes and Symmetry

20/21

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 3

Module leader
: David Wood

Credit value
: 15

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

This module investigates how solutions to systems of ODEs (in particular) change as parameters are smoothly varied resulting in smooth changes to steady states (bifurcations), sudden changes (catastrophes) and how inherent symmetry in the system can also be exploited. The module will be application driven with suitable reference to the historical significance of the material in relation to the Mathematics Institute (chiefly through the work of Christopher Zeeman and later Ian Stewart). It will be most suitable for third year BSc. students with an interest in modelling and applications of mathematics to the real world relying only on core modules from previous years as prerequisites and concentrating more on the application of theories rather than rigorous proof.

Module web page

Module aims

Understand how steady states can be dramatically affected by smoothly changing one or more parameters, how these ideas can be applied to real world applications and appreciate this work in the historical context of the department.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

1. Typical one-parameter bifurcations: transcritical, saddle-node, pitchfork bifurcations, Bogdanov-Takens, Hopf bifurcations leading to periodic solutions. Structural stability.

2. Motivating examples from catastophe and equivariant bifurcation theories, for example Zeeman Catastophe Machine, ship dynamics, deformations of an elastic cube, D_4-invariant functional.

3. Germs, equivalence of germs, unfoldings. The cusp catastrophe, examples including Spruce-
Budworm, speciation, stock market. Thom's 7 Elementary Catastrophes (largely through exposition rather than proof). Some discussion on the historical controversies.

4. Steady-State Bifurcations in symmetric systems, equivariance, Equivariant Branching Lemma,
linear stability and applications including coupled cell networks and speciation.

5. Time Periocicity and Spatio-Temporal Symmetry: Animal gaits, characterization of possible spatio-temporal symmetries, rings of cells, coupled cell networks, H/K Theorem, Equivariant Hopf
Theorem.

Further topics from (if time and interest):

Euclidean Equivariant systems (example of liquid crystals), bifurcation from group orbits (Taylor Couette), heteroclinic cycles, symmetric chaos, Reaction-Diffusion equations, networks of cells (groupoid formalism).

Learning outcomes

By the end of the module, students should be able to:

- Understand how observable (stable) solutions to systems of ODEs can be dramatically changed through a smooth change of parameter(s) through bifurcations and catastrophes.
- apply techniques from the whole module to examples, and understand the implications of the results, not limited to the idea that a dramatic change to solutions need not arise from a dramatic change to the equations (environment).
- Have a good grasp of the underlying theory, without necessarily being in a position to rigorously prove the main results. Appreciate how abstract material from previous core modules fit into it and can be applied.
- Appreciate the historical nature of the topics covered in the context of the Mathematics Institute.

Indicative reading list

- Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Guckenheimer/Holmes 1983
- Catastrophe Theory and its Applications, Poston and Stewart, 1978
- The Symmetry Perspective, Golubitsky and Stewart, 2002
- Singularities and Groups in Bifurcation Theory Vol 2, Golubitsky/Stewart/Schaeffer 1988
- Pattern Formation, an introduction to methods, Hoyle 2006.

Subject specific skills

See learning outcomes.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Seminars	10 sessions of 1 hour (7%)
Private study	110 hours (73%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group B1

	Weighting	Study time
In-person Examination	100%	
A 3-hour written exam.		

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Non-assessed assignments and support classes.

Past exam papers for MA3J3

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
  - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 3 of GG14 Mathematics and Statistics
    - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)