

MA3E1-15 Groups & Representations

20/21

Department

Warwick Mathematics Institute

Level

Undergraduate Level 3

Module leader

John Greenlees

Credit value

15

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

The concept of a group is defined abstractly (as set with an associative binary operation, a neutral element, and a unary operation of inversion) but is better understood through concrete examples, for instance:

-permutation groups

-matrix groups

-groups defined by generators and relations. All these concrete forms can be investigated with computers. In this module we will study groups by

-finding matrix groups to represent them

-using matrix arithmetic to uncover new properties. In particular, we will study the irreducible characters of a group and the square table of complex numbers they define. Character tables have a tightly-constrained structure and contain a great deal of information about a group in condensed form. The emphasis of this module will be on the interplay of theory with calculation and examples.

[Module web page](#)

Module aims

To introduce representation theory of finite groups in a hands-on fashion.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

Groups, subgroups. Examples (cyclic, dihedral, symmetric, alternating, matrix). Representations, equivalence of representations; the group ring, modules over the group ring. Sums of representations. Averaging and complements. Maschke's Theorem. Simple representations. Schur's Lemma. Traces and the character of a representation; Class functions.

Characters of fixed points. Characters of the representation $\text{Hom}(V,W)$. Hermitian inner products. Inner product of representations. Orthogonality of simple characters. Simple characters give an orthonormal basis of class functions. The character table. Row and column orthogonality.

Inflated representations. Kernels and normal subgroups from characters. Abelian groups. Abelianizations and the centre from the character table.

Tensor products. Symmetric and alternating squares. Restricted and induced representations and their characters. Frobenius reciprocity.

Integrality. The dimensions of simples divide the group order. Integral values of characters. Burnside's Theorem.

Learning outcomes

By the end of the module, students should be able to:

- understand matrix and linear representations of groups and their associated modules,
- compute representations and character tables of groups, and
- know the statements and understand the proofs of theorems about groups and representations covered in this module.

Indicative reading list

We will work through printed notes written by the lecturer.

A nice book that we shall not use is:

G James & M Liebeck, Representations and Characters of Groups, Cambridge University Press, 1993. Second edition, 2001. (ISBN: 052100392X).

Subject specific skills

Problem solving, calculation and experiment leading to clarity and proof. The importance of systematic and symmetrical organization. The complementarity of example and abstraction.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D1

	Weighting	Study time
Assigned work/tests	15%	
In-person Examination	85%	

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Marked coursework and exam feedback.

[Past exam papers for MA3E1](#)

Availability

Courses

This module is Optional for:

- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
 - Year 3 of G4G1 Discrete Mathematics
 - Year 3 of G4G1 Discrete Mathematics
- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
 - Year 3 of G300 Mathematics, Operational Research, Statistics and Economics
 - Year 4 of G300 Mathematics, Operational Research, Statistics and Economics
- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics

This module is Core option list B for:

- UMAA-GV17 Undergraduate Mathematics and Philosophy
 - Year 3 of GV17 Mathematics and Philosophy
 - Year 3 of GV17 Mathematics and Philosophy
 - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
 - Year 3 of G105 Mathematics (MMath) with Intercalated Year
 - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
 - Year 3 of G100 Mathematics

- Year 3 of G100 Mathematics
- Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
 - Year 3 of G100 Mathematics
 - Year 3 of G103 Mathematics (MMath)
 - Year 3 of G103 Mathematics (MMath)
 - Year 4 of G103 Mathematics (MMath)
 - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
 - Year 3 of G106 Mathematics (MMath) with Study in Europe
 - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 4 of UPA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
 - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
 - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
 - Year 3 of GG14 Mathematics and Statistics
 - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)

This module is Option list E for:

- USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
 - Year 3 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
 - Year 4 of G30D Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated)
 - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
 - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
 - Year 5 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)