MA359-15 Measure Theory

20/21

Department
: Warwick Mathematics Institute

Level
: Undergraduate Level 3

Module leader
: Oleg Pikhurko

Credit value
: 15

Module duration
: 10 weeks

Assessment
: Multiple

Study location
: University of Warwick main campus, Coventry

Description

Introductory description

The modern notion of measure, developed in the late 19th century, is an extension of the notions of length, area or volume. A measure m is a law which assigns a number to certain subsets A of a given space and is a natural generalization of the following notions: 1) length of an interval, 2) area of a plane figure, 3) volume of a solid, 4) amount of mass contained in a region, 5) probability that an event from A occurs, etc.

It originated in the real analysis and is used now in many areas of mathematics like, for instance, geometry, probability theory, dynamical systems, functional analysis, etc.

Given a measure m, one can define the integral of suitable real valued functions with respect to m. Riemann integral is applied to continuous functions or functions with `few` points of discontinuity. For measurable functions that can be discontinuous ``almost everywhere'' Riemann integral does not make sense. However it is possible to define more flexible and powerful Lebesgue's integral (integral with respect to Lebesgue's measure) which is one of the key notions of modern analysis.

[Module web page]

Module aims

To introduce the concepts of measure and integral with respect to a measure, to show their basic properties, and to provide a basis for further studies in Analysis, Probability, and Dynamical

Systems.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

The Module will cover the following topics: Definition of a measurable space and -additive measures, Construction of a measure form outer measure, Construction of Lebesgue's measure, Lebesgue-Stieltjes measures, Examples of non-measurable sets, Measurable Functions, Integral with respect to a measure, Lusin's Theorem, Egoroff's Theorem, Fatou's Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem, Product Measures and Fubini's Theorem. Selection of advanced topics such as Radon-Nikodym theorem, covering theorems, differentiability of monotone functions almost everywhere, descriptive definition of the Lebesgue integral, description of Riemann integrable functions, k-dimensional measures in n-dimensional spaces, divergence theorem, Riesz representation theorem, etc.

Learning outcomes

By the end of the module, students should be able to:

- To gain understanding of the abstract measure theory and definition and main properties of the integral.
- To construct Lebesgue's measure on the real line and in n-dimensional Euclidean space.
- To explain the basic advanced directions of the theory.

Indicative reading list

There is no official textbook for the course. As the main recommended book, I would suggest:
Cohn, D.L, Measure Theory, Second Edition, Birkhauser (2013). *
The list below contains some of many further books that may be used to complement the lectures.
Folland, G.b.: Real Analysis, Second Edition, Wiley (1999). *
Halmos, P. R.: Measure Theory, D. Van Nostrand Company Inc., Princeton, N.J. (1950) (Reprinted by Springer (1974)).
Kubrusly, C.S: Essentials of Measure Theory, Springer (2015). *
Loeb, P.A: Real Analysis, Birkhauser (2016). *
Royden, H. L. and Fitzpatrick, P.M: Real Analysis, Fourth Edition, Macmillan Publishing Company (2010).
Rudin, W.: Real and Complex Analysis, Third Edition, McGraw-Hill Book Company (1987).
Stein, E. M. and Shakarchi, R.: Real Analysis - measure theory, integration and Hilbert spaces. (Princeton Lectures in Analysis III) Princeton University Press (2005).

- = E-book avaliable from Warwick Library.

Subject specific skills

The students will acquire working knowledge of measure and integral with respect to a measure, including sigma-algebras, Dynkin classes, outer measures, the Lebesgue measure on the real line and a finite-dimensional Euclidean space, Lebesgue-Stieltjes measures, measurable and integrable functions, convergence theorems, Lusin's and Egoroff's Theorems, product measures and multivariable integrals, signed measures, Radon-Nikodym theorem, and Lp-spaces. This will form a basis for further studies in Ergodic Theory, Functional Analysis, Probability, etc.

Transferable skills

The students will be able to apply abstract notions in different contexts, in particular they will learn general techniques for studying parameters of large-scale systems arising in physical sciences. The students will also develop the ability to approach complex problems by approximating them by simpler ones.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (20%)
Tutorials	9 sessions of 1 hour (6%)
Private study	111 hours (74%)
Total	150 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D1

	Weighting	Study time
Assignments	15%	
In-person Examination	85%	

	Weighting	Study time

- Answerbook Gold (24 page)

Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	

- Answerbook Gold (24 page)

Feedback on assessment

Marked assignments and exam feedback.

Past exam papers for MA359

Availability

Courses

This module is Core for:

- Year 3 of UCSA-G4G3 Undergraduate Discrete Mathematics

This module is Core optional for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
  - Year 3 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int
  - Year 4 of G30E Master of Maths, Op.Res, Stats & Economics (Actuarial and Financial Mathematics Stream) Int

This module is Optional for:

- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
  - Year 3 of G4G1 Discrete Mathematics
- Year 4 of UCSA-G4G2 Undergraduate Discrete Mathematics with Intercalated Year
- Year 4 of USTA-G300 Undergraduate Master of Mathematics,Operational

Research,Statistics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)

This module is Core option list B for:

- Year 3 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- USTA-G301 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics (with Intercalated
    - Year 3 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
    - Year 4 of G30H Master of Maths, Op.Res, Stats & Economics (Statistics with Mathematics Stream)
- UMAA-GV17 Undergraduate Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
    - Year 3 of GV17 Mathematics and Philosophy
- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations
- Year 3 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)

This module is Core option list D for:

- Year 4 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- Year 1 of TMAA-G1PD Postgraduate Taught Interdisciplinary Mathematics (Diploma plus MSc)
- Year 1 of TMAA-G1P0 Postgraduate Taught Mathematics
- Year 1 of TMAA-G1PC Postgraduate Taught Mathematics (Diploma plus MSc)
- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
    - Year 3 of G105 Mathematics (MMath) with Intercalated Year
    - Year 5 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
    - Year 3 of G100 Mathematics
- UMAA-G103 Undergraduate Mathematics (MMath)
    - Year 3 of G100 Mathematics
    - Year 3 of G103 Mathematics (MMath)
    - Year 3 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
    - Year 4 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe

    - Year 3 of G106 Mathematics (MMath) with Study in Europe
    - Year 4 of G106 Mathematics (MMath) with Study in Europe
- Year 3 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
    - Year 3 of GF13 Mathematics and Physics
    - Year 3 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
    - Year 3 of FG31 Mathematics and Physics (MMathPhys)
- Year 4 of UPXA-GF14 Undergraduate Mathematics and Physics (with Intercalated Year)
- Year 4 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
    - Year 3 of Y602 Mathematics,Operational Research,Stats,Economics
- Year 4 of USTA-Y603 Undergraduate Mathematics,Operational Research,Statistics,Economics (with Intercalated Year)

This module is Option list B for:

- Year 1 of TMAA-G1PE Master of Advanced Study in Mathematical Sciences
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
    - Year 3 of GG14 Mathematics and Statistics
    - Year 3 of GG14 Mathematics and Statistics
- Year 4 of USTA-GG17 Undergraduate Mathematics and Statistics (with Intercalated Year)