

# MA209-6 Variational Principles

**20/21**

**Department**

Warwick Mathematics Institute

**Level**

Undergraduate Level 2

**Module leader**

Vasily Gelfreykh

**Credit value**

6

**Module duration**

10 weeks

**Assessment**

Multiple

**Study location**

University of Warwick main campus, Coventry

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## Description

### Introductory description

This module consists of a study of the mathematical techniques of variational methods, with applications to problems in physics and geometry. Critical point theory for functionals in finite dimensions is developed and extended to variational problems.

[Module web page](#)

### Module aims

To introduce the calculus of variations and to see how central it is to the formulation and understanding of physical laws and to problems in geometry.

### Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

This module is an introduction into mathematical techniques of variational methods, with applications to problems in Physics and Geometry. The basic problem in the calculus of variations is to minimise an integral which depends on a differentiable function and its derivatives. The module covers the following topics: a brief revision of critical points in finite dimension, the

mathematical set up of a variational problem, Euler-Lagrange equations for functionals of different types (including a derivation of these equations), a discussion of appropriate boundary conditions, first integrals of the Euler Lagrange equations, applications of variational principles to classical mechanics (including the least action principle) and optics (Fermat's principle). The theory is extended to constrained variational problems using Lagrange multipliers. The theory is illustrated by numerous examples.

## **Learning outcomes**

By the end of the module, students should be able to:

- At the conclusion of the course you should be able to set up and solve minimisation problems with and without constraints, to derive Euler-Lagrange equations and appreciate how the laws of mechanics and geometrical problems involving least length and least area fit into this framework.

## **Indicative reading list**

A useful and comprehensive introduction is:

R Weinstock, Calculus of Variations with Applications to Physics and Engineering, Dover, 1974.

Other useful texts are:

F Hildebrand, Methods of Applied Mathematics (2nd ed), Prentice Hall, 1965.

IM Gelfand & SV Fomin. Calculus of Variations, Prentice Hall, 1963.

The module will not, however, closely follow the syllabus of any book.

## **Subject specific skills**

At the conclusion of the module the student should be able to set up and solve various minimisation problems with and without constraints, to derive Euler-Lagrange equations and appreciate how the laws of mechanics and geometrical optics, as well as some geometrical problems involving least length and least area, fit into this framework.

## **Transferable skills**

The student will learn how the methods of mathematical analysis studied in the first and second year can be applied to model some simple real world phenomena such as the motion of a mechanical object or the shape of a ray of light. The student will see how mathematical methods can be used to restate some fundamental laws of Physics. The students will also learn some basics features of optimisation problems.

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## **Study**

## **Study time**

Type	Required
Lectures	15 sessions of 1 hour (25%)
Private study	45 hours (75%)
Total	60 hours

## Private study description

Review lectured material and work on set exercises.

## Costs

No further costs have been identified for this module.

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## Assessment

You do not need to pass all assessment components to pass the module.

### Assessment group B1

	Weighting	Study time
In-person Examination	100%	
<ul style="list-style-type: none"> <li>Answerbook Pink (12 page)</li> </ul>		

### Assessment group R

	Weighting	Study time
In-person Examination - Resit	100%	
<ul style="list-style-type: none"> <li>Answerbook Pink (12 page)</li> </ul>		

## Feedback on assessment

Exam feedback.

[Past exam papers for MA209](#)

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## Availability

## Courses

This module is Core for:

- Year 2 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
  - Year 2 of GF13 Mathematics and Physics
  - Year 2 of GF13 Mathematics and Physics
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)
  - Year 2 of FG31 Mathematics and Physics (MMathPhys)

This module is Optional for:

- Year 3 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 4 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
  - Year 2 of G1G3 Mathematics and Statistics (BSc MMathStat)
  - Year 3 of G1G3 Mathematics and Statistics (BSc MMathStat)
  - Year 4 of G1G3 Mathematics and Statistics (BSc MMathStat)
- Year 4 of USTA-G1G4 Undergraduate Mathematics and Statistics (BSc MMathStat) (with Intercalated Year)
- USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
  - Year 2 of GG14 Mathematics and Statistics
  - Year 2 of GG14 Mathematics and Statistics
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics

This module is Core option list B for:

- Year 3 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Core option list C for:

- Year 2 of UMAA-GV19 Undergraduate Mathematics and Philosophy with Specialism in Logic and Foundations

This module is Option list A for:

- UMAA-G105 Undergraduate Master of Mathematics (with Intercalated Year)
  - Year 2 of G105 Mathematics (MMath) with Intercalated Year
  - Year 3 of G105 Mathematics (MMath) with Intercalated Year
- UMAA-G100 Undergraduate Mathematics (BSc)
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 2 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics
  - Year 3 of G100 Mathematics

- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 2 of G100 Mathematics
  - Year 2 of G103 Mathematics (MMath)
  - Year 2 of G103 Mathematics (MMath)
  - Year 3 of G100 Mathematics
  - Year 3 of G103 Mathematics (MMath)
  - Year 3 of G103 Mathematics (MMath)
- UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
  - Year 2 of G106 Mathematics (MMath) with Study in Europe
  - Year 3 of G106 Mathematics (MMath) with Study in Europe
- Year 2 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 2 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 2 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 2 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- UMAA-G101 Undergraduate Mathematics with Intercalated Year
  - Year 2 of G101 Mathematics with Intercalated Year
  - Year 4 of G101 Mathematics with Intercalated Year

This module is Option list B for:

- UCSA-G4G1 Undergraduate Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
  - Year 2 of G4G1 Discrete Mathematics
- Year 2 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 2 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics
  - Year 2 of Y602 Mathematics, Operational Research, Stats, Economics