MA134-12 Geometry and Motion

20/21

Department
Warwick Mathematics Institute
Level
Undergraduate Level 1
Module leader
David Wood
Credit value
12
Module duration
10 weeks
Assessment
Multiple
Study location
University of Warwick main campus, Coventry

Description

Introductory description

When a particle moves in space, it traces out a curve. This is one of the simplest connections between geometry and motion. The motion contains more information than the curve traced out by the particle because the same curve can be traversed at different, possibly non-uniform, speeds (different motion). The length of the curve (a geometric property) is given by the integral (with respect to time) of the speed at which the curve is traversed. However, the length is evidently independent of the actual motion of the particle along the curve. This independence is established by means of the change of variables formula for integrals. Another connection between geometry and motion is provided by the relation between curvature and acceleration.
In high school, one learns how to integrate a function of one real variable. This course describes how to integrate vector-valued functions and functions of two and three real variables. In particular, the area of a surface and volume of a region (geometry) will be defined, as well as the circulation of a fluid around a closed curve (motion). The change of variables formula for two and three dimensional integrals will be (heuristically) derived; it involves a determinant and is somewhat more complicated than the one dimensional formula.
A section on particle mechanics will derive Kepler's Laws of planetary motion from Newton's second law of motion and the law of gravitation. The motion of the simple pendulum will also be discussed. This section reinforces the discussion of gradient flows in MA133 Differential Equations and introduces the notion of conserved quantities.

Module web page

Module aims

This module aims to indicate to students how intuitive geometric and physical concepts such as length, area, volume, curvature, mass, circulation and flux can be translated into mathematical formulas. It also aims to teach the practical calculation of these formulas and their application to elementary problems in particle and fluid dynamics. The importance of conserved quantities in mechanics is also highlighted.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

An introduction to curves and the notions of parametrisation, tangent vectors and curvature, with applications in mechanics. A first look at differentiation and integration for functions of several variables applied to both geometric and physical concepts, including vector fields, surface and line integrals, work and potential energy.

Learning outcomes

By the end of the module, students should be able to:

- understand basic notions from particle mechanics including momentum (linear and angular), force, work, energy (potential and kinetic), Newton's laws of motion, Newton's law of gravity, conservation laws. Students should also be able to apply all these principles to elementary problems from mechanics, including central force theory (including, but not restricted to, planetary motion) and the simple pendulum.
- parametrise simple curves and surfaces, such as conic sections, helix, surface of revolution (including sphere, cylinder, paraboloid and torus), in cartesian and other coordinates, including polar, spherical polar and cylindrical coordinates.
- calculate lengths and curvatures of curves in 3-space and demonstrate that length is independent of parametrisation.
- understand and be able to calculate line, surface and volume integrals with respect to various coordinate systems. This includes change of variables and change of order of integration in repeated integrals. Please note that in the examination, no formula sheets will be provided.
- to be able to determine whether a vector field is conservative and to calculate its potential when it is.
- apply all these techniques to elementary problems from fluid dynamics (mass, work, circulation and flux) and geometry (area and volume).

Indicative reading list

G.B. Thomas et al., Calculus and Analytic Geometry, Addison-Wesley. The course is concerned with only the later chapters of this massive book. However, the earlier chapters are relevant to

other first year courses and even contain A-Level material from a different perspective. Any edition of this book is appropriate. You may be able to buy a cheap copy through Amazon.
F.J. Flannigan and J.L. Kazdan, Calculus Two, Springer-Verlag. Again, the earlier chapters of this book are relevant to other first year courses.
J.E. Marsden and A.J. Tromba, Vector Calculus, Freeman. This book is more advanced than Calculus Two and is useful for the second year courses on Vector Analysis and Differentiation.

Subject specific skills

This module will introduce students to connections between geometry, calculus and physical modelling, providing them with the skills to convert between reasoning physically and geometrically. Further, the understanding of and ability to manipulate concepts from multivariable calculus first developed here is crucial to understand many of the mathematical models used to describe the world around us.

Transferable skills

Along with developing problem solving skills and logical reasoning, a key aim of this module is building confidence in visualising and sketching geometric objects, enabling the development of spatial awareness and visual communication skills.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (25%)
Tutorials	9 sessions of 1 hour (8%)
Private study	81 hours (68%)
Total	120 hours

Private study description

Review lectured material and work on set exercises.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Assessment group D1

	Weighting	Study time	Eligible for self-certification
Written homework	15%		Yes (extension)
In-person Examination	85%		No

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit exam	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Marked coursework/homework and exam feedback.

[Past exam papers for MA134]

Availability

Courses

This module is Core for:

- Year 1 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)
  - Year 1 of G100 Mathematics
  - Year 1 of G103 Mathematics (MMath)
- Year 1 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 1 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 1 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 1 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 1 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 1 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 1 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year

- Year 1 of UMAA-G101 Undergraduate Mathematics with Intercalated Year

This module is Optional for:

- Year 1 of USTA-G300 Undergraduate Master of Mathematics,Operational Research,Statistics and Economics
- Year 1 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 1 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 1 of USTA-Y602 Undergraduate Mathematics,Operational Research,Statistics and Economics

This module is Option list A for:

- Year 1 of UCSA-G4G1 Undergraduate Discrete Mathematics
- Year 1 of UCSA-G4G3 Undergraduate Discrete Mathematics