

MA106-12 Linear Algebra

20/21

Department

Warwick Mathematics Institute

Level

Undergraduate Level 1

Module leader

David Wood

Credit value

12

Module duration

10 weeks

Assessment

Multiple

Study location

University of Warwick main campus, Coventry

Description

Introductory description

Many problems in maths and science are solved by reduction to a system of simultaneous linear equations in a number of variables. Even for problems which cannot be solved in this way, it is often possible to obtain an approximate solution by solving a system of simultaneous linear equations, giving the "best possible linear approximation".

The branch of maths treating simultaneous linear equations is called linear algebra. The module contains a theoretical algebraic core, whose main idea is that of a vector space and of a linear map from one vector space to another. It discusses the concepts of a basis in a vector space, the dimension of a vector space, the image and kernel of a linear map, the rank and nullity of a linear map, and the representation of a linear map by means of a matrix.

These theoretical ideas have many applications, which will be discussed in the module. These applications include:

Solutions of simultaneous linear equations.

Properties of vectors.

Properties of matrices, such as rank, row reduction, eigenvalues and eigenvectors.

Properties of determinants and ways of calculating them.

[Module web page](#)

Module aims

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Outline syllabus

This is an indicative module outline only to give an indication of the sort of topics that may be covered. Actual sessions held may differ.

- The vector space \mathbb{R}^n , including a geometric description of vector addition in \mathbb{R}^2 .
- Fields. Definition of a vector space V over a field. The space spanned by a subset of V . Linear dependence and independence. Bases. Dimension. Subspaces. Dual spaces and dual bases.
- Linear maps $f:V \rightarrow W$. Isomorphism of vector spaces. Any n -dimensional vector space over F is isomorphic to \mathbb{R}^n . Examples of linear maps, including differentiation and integration as linear maps on spaces of functions or polynomials.
- Matrices. Algebraic operations on matrices. Reduction of a matrix using row and column operations. Application to the solution of linear equations. Rank. Row rank = Column rank.
- The relation between linear maps and matrices. the matrix of a linear map with respect to a given basis. Change of basis changes A to PAQ^{-1} . The kernel and image of $f:V \rightarrow W$. The rank and nullity of f .
- Determinants, defined by $\sum_{\sigma \in S_n} \text{sign } \sigma (\prod a_{i,\sigma(i)})$. $\text{Det}(AB) = \text{Det}(A)\text{Det}(B)$ (proof either in general or in the cases $n=1,2,3$). Submatrices, minors, cofactors, the adjoint matrix. Rules for calculating determinants. The inverse of a matrix. $Ax=0$ has non-zero solution if and only if $\text{det}(A) \neq 0$. Determinantal rank.
- Eigenvalues and eigenvectors. Definition and examples. Their geometric significance. Diagonalisation of matrices with distinct eigenvalues.
- Inner product spaces and isometries. Euclidean spaces. Orthogonal transformations and matrices.

Learning outcomes

By the end of the module, students should be able to:

- Understand and demonstrate knowledge of vector spaces, fields, linear dependence and independence, bases and dimension.
- Understand linear transformations and be able to show examples of linear maps such as differentiation and integration as linear maps on spaces of functions or polynomials.
- Be proficient at matrix manipulation, reduction of a matrix using row and column operations and be able to apply to finding solutions to linear equations.
- Be able to compute determinants for general n by n matrices, compute cofactors and adjoint matrices and understand the implications of doing this to solving sets of linear equations.
- Be able to compute eigenvalues and eigenvectors of matrices and understand their geometric significance. Be able to diagonalize matrices with distinct eigenvalues.

Indicative reading list

David Towers, Guide to Linear Algebra, Macmillan 1988.

Howard Anton, Elementary Linear Algebra, John Wiley and Sons, 1994.

Paul Halmos, Linear Algebra Problem Book, MAA, 1995.

G Strang, Linear Algebra and its Applications, 3rd ed, Harcourt Brace, 1988.

Subject specific skills

To provide a working understanding of matrices and vector spaces for later modules to build on and to teach students practical techniques and algorithms for fundamental matrix operations and solving linear equations.

Transferable skills

Students will acquire key reasoning and problem solving skills which will empower them to address new problems with confidence.

Study

Study time

Type	Required
Lectures	30 sessions of 1 hour (25%)
Tutorials	6 sessions of 30 minutes (2%)
Private study	87 hours (72%)
Total	120 hours

Private study description

Working on assignments, going over lecture notes, text books, exam revision.

Costs

No further costs have been identified for this module.

Assessment

You do not need to pass all assessment components to pass the module.

Students can register for this module without taking any assessment.

Assessment group D2

	Weighting	Study time	Eligible for self-certification
Assessment (non-Maths students) weekly, summative, assignments	15%		Yes (waive)
In-person Examination Exam	85%		No

- Answerbook Pink (12 page)

Assessment group R

	Weighting	Study time	Eligible for self-certification
In-person Examination - Resit exam	100%		No

- Answerbook Pink (12 page)

Feedback on assessment

Marked assignments, face to face supervisions.

[Past exam papers for MA106](#)

Availability

Courses

This module is Core for:

- Year 1 of USTA-G302 Undergraduate Data Science
- Year 1 of USTA-G304 Undergraduate Data Science (MSci)
- Year 1 of UCSA-G4G1 Undergraduate Discrete Mathematics
- Year 1 of UCSA-G4G3 Undergraduate Discrete Mathematics
- Year 1 of USTA-G300 Undergraduate Master of Mathematics, Operational Research, Statistics and Economics
- Year 1 of UMAA-G100 Undergraduate Mathematics (BSc)
- UMAA-G103 Undergraduate Mathematics (MMath)

- Year 1 of G100 Mathematics
- Year 1 of G103 Mathematics (MMath)
- Year 1 of UMAA-G106 Undergraduate Mathematics (MMath) with Study in Europe
- Year 1 of UMAA-G1NC Undergraduate Mathematics and Business Studies
- Year 1 of UMAA-G1N2 Undergraduate Mathematics and Business Studies (with Intercalated Year)
- Year 1 of UMAA-GL11 Undergraduate Mathematics and Economics
- Year 1 of UECA-GL12 Undergraduate Mathematics and Economics (with Intercalated Year)
- Year 1 of UMAA-GV17 Undergraduate Mathematics and Philosophy
- Year 1 of UMAA-GV18 Undergraduate Mathematics and Philosophy with Intercalated Year
- Year 1 of UPXA-FG33 Undergraduate Mathematics and Physics (BSc MMathPhys)
- Year 1 of UPXA-GF13 Undergraduate Mathematics and Physics (BSc)
- UPXA-FG31 Undergraduate Mathematics and Physics (MMathPhys)
 - Year 1 of GF13 Mathematics and Physics
 - Year 1 of FG31 Mathematics and Physics (MMathPhys)
- Year 1 of USTA-G1G3 Undergraduate Mathematics and Statistics (BSc MMathStat)
- Year 1 of USTA-GG14 Undergraduate Mathematics and Statistics (BSc)
- Year 1 of UMAA-G101 Undergraduate Mathematics with Intercalated Year
- Year 1 of USTA-Y602 Undergraduate Mathematics, Operational Research, Statistics and Economics